A WAVE ANALYZER EMPLOYING VARIBLE SPEED MAGNETIC TAPE

James Vernon Haley
and
Jeremiah E. Lenihan

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JAMES V. HALEY, Lieutenant (junior grade), U. S. Navy
B.S., U.S. Naval Academy (1949)

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SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE
DEGREE OF NAVAL ENGINEER
at the
MASSACHUSETTS INSTITUTE OF
TECHNOLOGY

June, 1954

Cambridge, Massachusetts
May 24, 1954

Professor L. F. Hamilton
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge, Massachusetts

Dear Professor Hamilton:

In accordance with the requirements for the degree of Naval Engineer, we submit herewith a thesis entitled "A Wave Analyzer Employing Variable Speed Magnetic Tape".

Respectfully,

James V. Haley
Lieutenant (junior grade),
U. S. Navy

Jeremiah E. Lenihan
Lieutenant,
U. S. Navy

~~25061~~
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A WAVE ANALYZER EMPLOYING VARIABLE SPEED MAGNETIC TAPE

by

James V. Haley, Lieutenant (junior grade) U. S. Navy
Jeremiah E. Lenihan, Lieutenant, U. S. Navy

Submitted to the Department of
Naval Architecture and Marine Engineering
May 24, 1954
in partial fulfillment of the requirements for the
degree of Naval Engineer

ABSTRACT

The object of this thesis is to make a preliminary investigation of a new type wave analyzer. The proposed wave analyzer would employ a variable speed magnetic tape to effect a multiplication in frequency. The resultant analysis would be made with a constant-percentage resolution.

A basic analyzer theory has been formulated. Specific theoretical results include: (1) determination of two multiplier speed-time relationships, a minimum-analysis-time solution, and an equal-analysis-sample solution, (2) determination of the required length of sample tape which is found to be a function of the desired percentage resolution, and (3) evaluation of the response of a simple selective network to a frequency excitation which varies very nearly linearly with time while within the pass-band. Dynamic amplitude and frequency distortion will occur as a result of the variable frequency excitation. It is shown that these analyzer errors can be predicted by use of a single parameter.

An experimental investigation of the analyzer principles was attempted using magnetic tape wound around a disc. The speed of the disc was controlled by a Ward-Leonard system. Other experiments were made with the speed of the disc slowing down due to its own damping. Experimental results were limited by: (1) variation in the distance between the reproduce head and the magnetic tape due to the eccentricity of the disc, and (2) inability to control accurately the speed of the disc.

Within the pass-band the deviation from a linear sweep for both multiplier functions is less than 1/Q. It is concluded that linear-frequency-sweep theory is applicable to the proposed analyzer. Furthermore, it is concluded that equal sample analysis is a desirable feature in order to avoid a possible time-distribution ambiguity in measurements. However, the required multiplication function unfavorably affects total analysis time requirements and analyzer complexity.

Thesis Supervisor: Thomas F. Jones, Jr.
Title: Assistant Professor of
Electrical Engineering

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The authors thank Professor T. F. Jones, Jr., ever burdened by a heavy schedule in his own department, for his supervision and counsel. The basic concept of this thesis originated with Dr. J. W. Horton, Chief Research Consultant, U. S. Navy Underwater Sound Laboratory, New London, Connecticut. Dr. Horton also was helpful in arranging for equipment. The authors are obligated to the U.S. Navy Underwater Sound Laboratory for equipment necessary for the progress of the thesis. The Acoustics Laboratory at M.I.T. assisted in providing additional equipment and suggestions. The skill of our typist, Miss Frances Doherty, is evident in the following pages.

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CHAPTER I

INTRODUCTION

The objective of this thesis is the investigation of a new type of electronic wave analyzer.[*] The proposed analyzer would employ a variable-speed magnetic medium to effect a multiplication in frequency. This multiplication in frequency would be employed in a manner analagous to the addition of frequencies in the conventional wave analyzer. As in such analyzers components of various frequencies are measured by sweeping a derived component across a fixed selective network. The advantage of frequency multiplication over frequency addition is that the resultant analysis may be made with a constant-percentage resolution[**] rather than with a percentage resolution which varies between wide limits.

The device is intended to assist in the processing and analysis of data available in the form of magnetically recorded signals. The data will have been recorded at some constant speed. An automatic analysis will be effected by continuously varying the reproduce speed of the magnetic medium. Minimum analysis time is desired. Hence, the reproduce speed would be varied at the highest rate consistent with the limitations imposed by: (1) the response of the fixed selective network to an input of varying frequency, and (2) the specification of certain

---

[*] This analyzer was proposed to the authors by Doctor J. W. Horton, Chief Research Consultant, U.S. Navy Underwater Sound Laboratory, Fort Trumbull, New London, Connecticut.

[**] By resolution is meant the ability to distinguish between two closely adjacent frequency components. Percentage resolution is defined as resolution (measured in cycles per second) expressed as a percentage of mid-band frequency of the selective network.

desired analyzer characteristics which will be proposed in Chapter II-A. Automatic operation coupled with minimum analysis time would do much to alleviate the laborious and time-consuming analysis which is now necessary using commercially available wave analyzers.

Ideally, the analyzer should be able to consider and process signals lying within the frequency spectrum from 1 cycle to 100 kilocycles. Practically, certain characteristics of present-day recording techniques will limit the analysis spectrum-band. It is anticipated that at least two frequency-bands will be required to complete the analysis.

With the ever-increasing desire to investigate phenomena from a spectrum analysis consideration there is a decided need for equipment which would facilitate such measurements. In addition, there is a real need in the field of underwater sound for a constant percentage resolution analyzer which would incorporate automatic operation and rapidity of measurement. It is believed that similar needs exist in other fields.

In a sense the proposed analyzer will be composed of components which have been investigated at some length and which are in common use today. These components will be arranged, and will be caused to function in a manner to accomplish a specific desired result: spectrum analysis. This thesis will emphasize the inter-relationship between these components in order to accomplish this purpose, and will investigate the limitations which various components put on the problem and the system as a whole. It is not our intention to build a prototype wave analyzer. The thesis will attempt to answer as far as possible the question of the feasibility of the proposed analyzer.

The Basic Analyzer

The proposed analyzer could be divided into four functional sub-

divisions: (1) a frequency multiplier, (2) a fixed selective network, (3) an indicator, and (4) a control and programming section. A postulated arrangement is presented in Figure 1.1.

The signal to be analyzed is recorded at some constant speed on either magnetic tape or on the outer edge of a magnetic disc. This closed loop of predetermined length is constrained by the frequency multiplier to follow an appropriate speed-time relationship, $N(t)$.[*] Such a multiplication might be achieved by: (1) continuously varying the speed of a magnetic disc, (2) wrapping magnetic tape around the periphery of a non-magnetic disc which proceeds at a variable speed, and (3) transporting a constant speed magnetic medium over a reproduce head array which rotates in accordance with $N(t)$.

Following the multiplier a reproduce system converts the recorded signals to an electrical excitation which sweeps across a fixed selective network. Selectivity is provided by a single, high - Q, band-pass filter per frequency-band. Next the signals proceed to the indicator, pass through appropriate averaging circuits, and finally are presented in the form of a permanent visual record. If simultaneous analysis of the entire spectrum is desired, one graphic-recorder channel per frequency-band is required.

Note that the speed-time relationship, $N(t)$, is a fundamental link between three main analyzer sections. An appropriate tie-in must exist between the multiplier and the indicator. There is a possibility that the averaging circuits might also be a function of $N(t)$. It appears that the control and programming section will require use of servomechanisms combined with suitable electronic control circuits.

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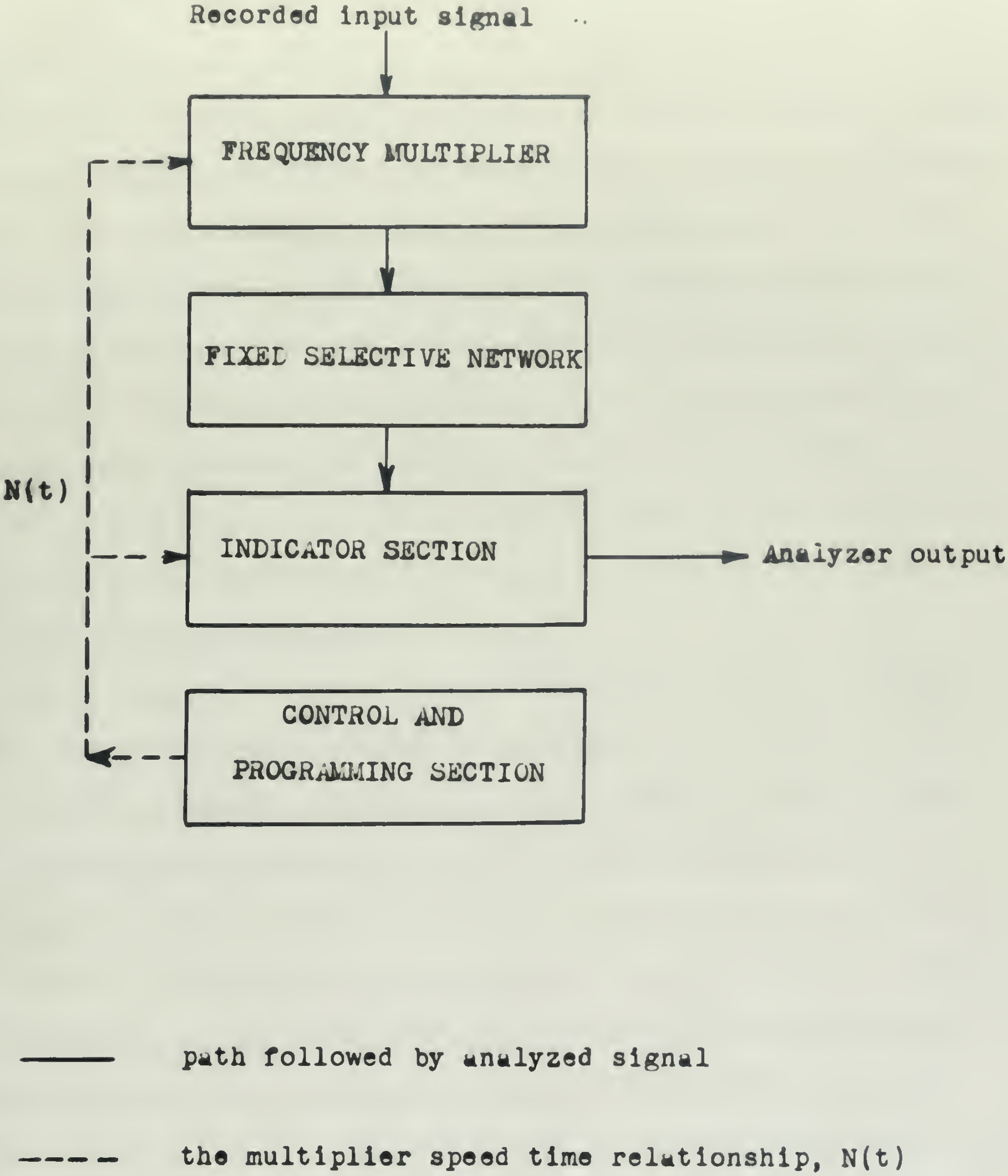
[*] Appendix A provides a table of symbols and definitions.

FIGURE 1.1

BASIC ANALYZER BLOCK DIAGRAM


Recorded input signal
FREQUENCY MULTIPLIER
FIXED SELECTIVE NETWORK
INDICATOR SECTION
Analyzer output
CONTROL AND PROGRAMMING SECTION
N(t)
—— path followed by analyzed signal
---- the multiplier speed time relationship, N(t)

Wave Analyzers In General

A thorough survey of available technical literature indicates that the proposed wave analyzer would have several unique characteristics. The basic novelty exists in the frequency multiplier. The variable speed magnetic medium effects a constant-percentage resolution analysis.

Beranek[4] presents a comprehensive study of the many different types of apparatus utilized to analyze a complex noise into a spectral distribution. Jastram and McCouch[13] have discussed the design requirements for a wave analyzer capable of measuring noise spectra in the video-frequency range. Their paper includes a discussion on the response of a resonant system to a non-periodic function. Another paper which concentrates on the design features of a spectrum analyzer was published recently by Soanes.[19] He discusses the increased number of problems associated with low frequency analysis where the time of one cycle becomes comparable with the total time available for the analysis.

Actually the plan to design a wave analyzer employing a frequency multiplying device with a fixed selective system is not a new one. In 1924 Sacia[18] proposed a non-automatic analyzer which was very accurate, but which required a great deal of analysis time. The sound waves were recorded on a strip of film, joined into an endless band, and then picked off with the aid of a photo-cell. Following the photo-cell was an amplifier containing a tuned electrical circuit. The speed of the film was varied so that the different partial tones of the sound wave were recorded singly, and an analysis was obtained. Similarly Barber and Ursell[1,2] have designed a wave analyzer used for examining spectra of ocean waves.

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[4] Subscript numerals refer to similarly numbered entries in the Bibliography.

The record in the form of a variable-area black trace on a white background is wrapped around the periphery of a wheel. The wheel is made to revolve, and an electrical output is obtained from an optical device which views the record through a narrow slit close to the wheel. This output, a continual repetition of the recorded trace, is made to drive a sharply resonant vibration galvanometer. The successive resonances are recorded by a pen on moving paper. It is noted that its analysis time is about five minutes per octave; this is considered excessive for our wave analyzer.

Apparently the technique of designing a workable automatic wave analyzer has just recently been established.* As recently as 1949 Barber[3] suggested a slow mechanical drive combined with continuous recording of analyzer energy output in order to expedite the spectrum analysis. His paper includes a detailed study on the optimum performance of such an analyzer.

Wave Analyzers Employing Magnetic Media

Magnetic recording processes are rapidly being applied in the instrumentation field. Uses include pulse systems, and carrier systems. Furthermore, conventional magnetic recorders employing high-frequency bias are being used for recording a band-width within the range of 100 cycles to 100 kilocycles.[27]

Certain research and industrial applications of magnetic tape have been previously restricted by limitations of the medium itself.[26] Recently tape manufacturers have made enormous progress in the quality of their products which are designed for research applications.[23] These advances have been stimulated largely by the exacting demands of the

---

*By automatic we mean that the analyzer is self acting, and that a continuous, permanent record of its output is made available.

of the computer field for improved magnetic tape memory devices.

Why use magnetic recording and reproduction techniques in wave analyzers? The answer to this question is contained within two characteristic advantages of magnetic media. First, there are those general functions characteristic of magnetic recording: (1) recording and storage of signals for reproduction at will, and (2) improved signal-to-noise ratio.

An excessive length of analysis time is required using conventional methods. It would be desirable to accelerate the rate of scanning. This can be accomplished by use of magnetic media. Its second characteristic advantage is contained in the following discussion. A magnetic pattern is impressed in a medium traveling at a given velocity. If it is then passed over a pickup device at the same velocity, a characteristic wave length will be observed which is equal to the wave length originally recorded on the tape. Next, if the medium velocity is doubled over the recording velocity, the reproduce head sees two wave lengths in the same length of time it originally saw only one. As a result, the frequency obtained during play-back is twice that which was originally recorded. This characteristic can be expressed in a general form as

$$\frac{S}{S_r} = \frac{f_b}{f_a} = N$$

where

S = the instantaneous reproduce speed.

S_r = the original recording speed.

f_a = the recorded frequency component.

f_b = the frequency observed at the reproduce head when the magnetic medium is traveling at speed S.

N = the instantaneous reproduce speed or reproduce frequency ratio

This multiplication feature allows the original frequency spectrum to be converted to an equivalent spectrum in a higher frequency range. A wider band-width analyzing filter results with its subsequent lower build-up time.[8] The effective band-width of the analyzer filter, when referred to the original frequency spectrum, is the actual filter band-width divided by the reproduce speed ratio, N.

For any fixed reproduce speed, S, the rate of scanning is increased, and total analysis time is decreased. This comes from the requirement that for linear sweep excitation the ratio of sweeping is directly proportional to the square of the bandwidth.[19]

There are several automatic wave analyzers which employ a magnetic medium to effect a multiplication of frequency spectra. Two such commercial analyzers are Bell Telephone Laboratories' Sound Spectograph,[23,20] and Kay Electric Company's Vibralyzer.[29] These devices only make limited use of this important multiplication feature. In each case the medium reproduce speed is maintained at some constant value, which is usually only 3 to 5 times the record speed. Instead of making N a function of time, as is the case for the proposed analyzer, the analysis is obtained by use of a heterodyne process. No known analyzer achieves frequency selectivity by use of a continuously varying reproduce speed. In this respect the proposed analyzer is unique.

The Sound Spectrograph, also known commercially as the Sona-Graph, is a wave analyzer which produces a permanent visual record showing the distribution of energy in both frequency and time. Audio sounds up to 8000 cps can be studied. A typical recorded sample length is from 2 to 3 seconds duration. The signal to be analyzed is recorded on a loop of magnetic tape which is mounted on a rotating disc. Analysis

results are marked on electrically sensitive paper. This record is mounted on a cylindrical drum, which is mechanically coupled to the tape loop.

The Vibralyzer furnishes a frequency analysis of low-frequency sounds in the range from 5 to 4400 cps. The signal to be analyzed is recorded on the outer edge of a magnetic disc. This closed-loop signal is scanned with each revolution of a recording drum. The output is graphically presented on a sheet of facsimile-type paper for a permanent visual record. Information is presented regarding time, frequency, and amplitude of the recorded signal.

Variable Frequency Excitation of Selective Networks

Preliminary investigation indicates that we will be concerned with the response of linear selective networks to driving functions in which the frequencies vary other than linearly with time. There is very little information available on such driving functions. The response of a LCR circuit to a logarithmic frequency-sweep driving function has been investigated by Penfield.[17] This theoretical development resulted from a lengthy, graphical evaluation of a convolution integral equation. Unfortunately, this study only provides information for a specific case out of the yet-unexplored family of response curves which exist for a logarithmic-frequency-sweep excitation.

On the other hand, there is a great deal of information available on the response of selective networks to linear-frequency-sweep excitation. A review of the technical literature will be provided below. Briefly these studies can be summarized as follows. The response of a selective network will be nearly identical to its steady-state response provided the sweep rate is very low. As the rate of sweeping is increased, circuit transients make the response appreciably different from that obtained

with slow sweep rates. There will exist a dynamic, amplitude-depression distortion. A frequency-displacement distortion will also be associated with the maximum response. At the same time the effective band-width is increased over its nominal value. This decreases the effective value of Q. Furthermore, a ringing phenomena following the resonant response peak can result. This can be characterized as the beat-tone between the damped oscillation of the selective circuit and the sweep frequency signal. The presence of these secondary maxima can completely mask the detection of weak components in the vicinity of a strong component.

Many authors have studied the response of resonant systems to excitation of a frequency varying linearly with time. Clavier[5] sought the mathematical condition for which the dynamic response is nearly the same as the static response. His solution indicates the need for very slow exploration speeds. Lewis[14] investigated the case of a mechanical shaft of linearly varying speed of rotation. His graphical solution involves fresnel integrals of complex argument which are not known to be tabulated to any extent even today. Hok[10] studied the response of a narrow-band resonant system. Hamilton's[9] solution to the problem involves the fourier analysis of the input and output spectra. Barber and Ursell[2] have presented a solution for the response of an oscillatory system to a tone whose frequency slowly increases or decreases. It is shown that the form of the response is very complicated, but that the variation of amplitude near resonance depends upon a single parameter involving the constants of the apparatus. Marique[15] investigated a sawtooth varying frequency excitation. He shows that the response is composed of two terms: one arising during each sweep, the other resulting from the preceding sweep. Soanes[19,20] completes the general problem by discussing the effects of starting the sweep in or near the pass-band. In addition,

he provides an excellent review of many different authors' approaches to the general problem of linear sweep excitation. In 1935, Meyer[16] provided an earlier correlation of similar studies. Ekstein and Schiffman[8] recently investigated the response of a linear network to an input with linearly variable frequency as obtained in sweep frequency testing.

Scope of the Thesis

Time and economic considerations necessitate rather narrow limitations in the scope of the thesis. Obviously we cannot hope to design and construct a complete wave analyzer. Furthermore, there is considerable material available in the literature on magnetic reproduce systems, selective networks, averaging circuits, indicators, servomechanisms, and control circuits. This information will be readily available to the future designer. Hence, the thesis will concentrate on the unique features of the analyzer: the frequency multiplier and its associated speed-time relationship. These features raise certain questions which are not specifically answered in the literature:

(1) What multiplier speed-time relationships are available which result in tolerable limits of distortion, minimum analysis time, and desired analyzer characteristics?

(2) What is the proper relationship between desired percentage resolution and minimum analysis time per frequency band?

(3) What limitations do magnetic recording characteristics impose upon the overall analyzer?

(4) How many analysis frequency-bands should be used?

(5) What length of sample is necessary for proper analysis?

(6) What is the influence of N(t) upon the averaging, indicating, and programming sections?

CHAPTER II-A

DEVELOPMENT OF ANALYZER THEORY

This chapter introduces a basic theory applicable to the proposed analyzer. The analysis is guided by specific questions raised in Chapter I.

Two fundamental requirements for any automatic wave analyzer are: (1) to provide the greatest possible resolution of frequency components, and (2) to complete the analysis in the minimum possible time. Note that these are two conflicting aims. The smaller the pass-band of the selective network, the greater the resolution.[3,19] However, accurate output measurements are obtainable only if each component remains within this smaller pass-band for a longer period of time. Furthermore, it is possible (as is the case for this particular analyzer) that certain additional requirements, other than the characteristics of the selective network, will influence the determination of $N(t)$.

The Basic Selective System

The basic frequency selective system consists of a frequency multiplier, and a fixed selective network. Assume that the system input can be represented by a series of n discrete sinusoidal components, each of frequency f_{an}.* The output of the multiplier is then a series of frequency modulated components, $f_{bn}(t)$. The multiplier function can be expressed in terms of the instantaneous magnetic-tape reproduce speed, $S(t)$, and the fixed recording speed, S_r.

$$N(t) = \frac{S(t)}{S_r} \qquad (1)$$

---

*Where subscript n takes on values 0, 1, 2, ...

A fundamental multiplier relationship exists:

$$f_{bn}(t) = f_{an}N(t) = f_{an}\,\frac{S(t)}{S_r} \qquad (2)$$

It is appropriate to examine the physical operations involved in this system. A series of discrete components, f_{an}, have been recorded on magnetic tape at a constant recording speed, S_r. These are shown in Figure 2.1. It is logical to assume that the multiplier speed-time relationship will be in the form of a continuous, decaying function. Figure 2.2 is a sketch of such an assumed $N(t)$. Each recorded component, f_{an}, is multiplied by $N(t)$. Hence, the output of the multiplier is a family of decaying components, $f_{bn}(t)$, similar to those represented in Figure 2.3. These components sweep across the narrow-band selective network which ideally rejects all components lying outside the shaded area of Figure 2.3

The phenomena of frequency sweep must be considered from two different points of view. First, for any fixed input frequency (e.g. f_{a1}), its derived component sweeps across the pass-band in a decaying fashion. For example, when

$$t = t_1, \quad f_{b1} = f_u$$

and, when

$$t = t_2, \quad f_{b1} = f_L$$

where f_u and f_L are the upper and lower cut-off frequencies for the selective network. On the other hand, derived components of lower-frequency recorded signals (e.g. f_{a1}) are swept across the selective network before those derived components which are associated with higher-frequency recorded signals. (e.g. f_{a2}).

The response of selective networks to linear-frequency-sweep excitation has been thoroughly investigated. (See discussion in Chapter I). Examination of our basic system fails to indicate that $N(t)$ will necessarily

Figures 2.1 - 2.3

BASIC PHYSICAL CONSIDERATIONS OF MULTIPLIER ACTION

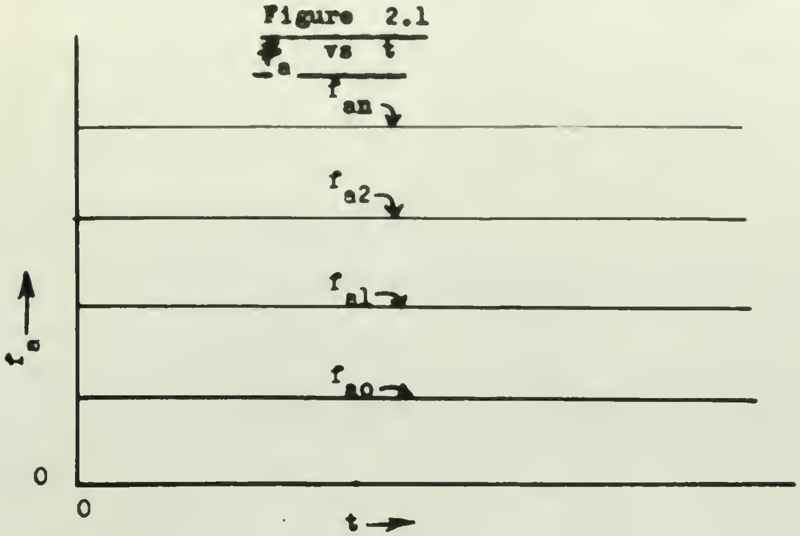

Figure 2.1

f_a vs t

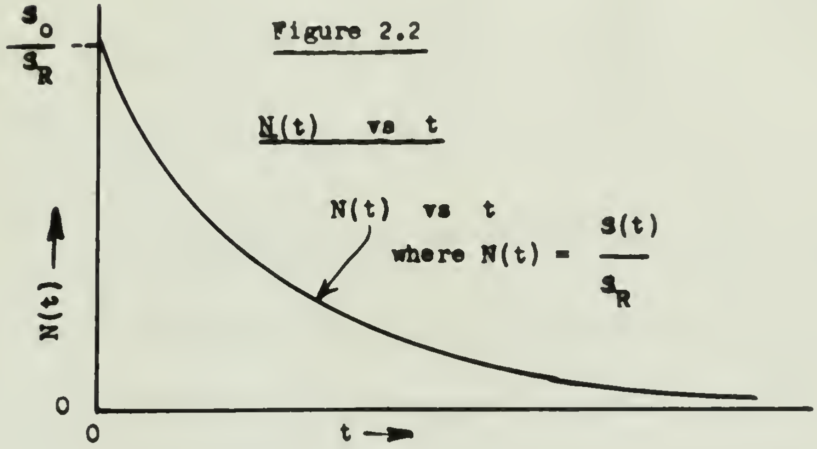

Figure 2.2

N(t) vs t

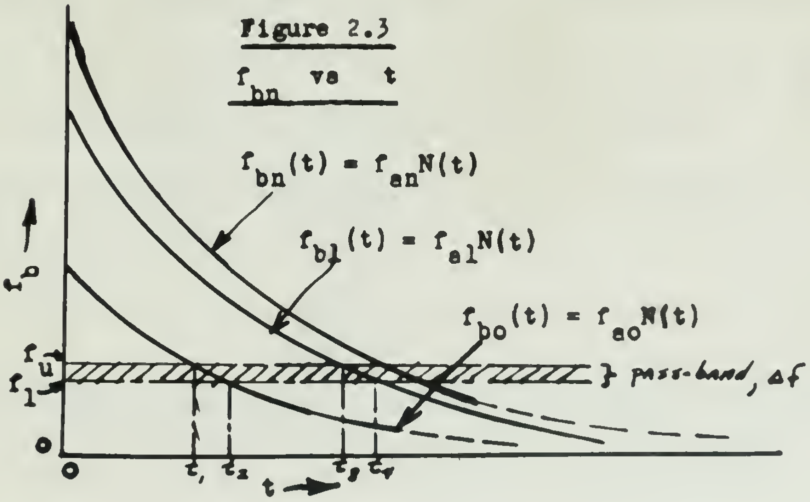

Figure 2.3

f_{bn} vs t

be a linear relationship. Nevertheless, this wave analyzer typically contains a narrow pass-band in order to obtain maximum frequency resolution. As a result, N(t) might properly be approximated as a linear sweep within this narrow frequency-band. Available linear-frequency-sweep research could then be applied as an aid in properly evaluating N(t).

The Multiplier Speed-Time Relationship

In Appendix B three theoretical multiplier speed-time relationships are derived by a qualitative approach.

First, the minimum-analysis-time solution is determined. It can be expressed in the form

$$S(t) = S_o e^{-ct} \tag{11b}$$

where S_o = the value of S at t equals 0.

$c = \frac{(\Delta f)^2}{Kf_u}$

Δf = the width of the resonant response curve at the cut-off frequencies

f_u = the upper cut-off frequency of the selective network.

K = a pure numeric (cycles)

For this solution each derived component, f_{bn}, remains within the pass-band the minimum time for acceptable analysis. This condition is based on the requirement that the output of the selective system remains within certain tolerable limits of amplitude and frequency distortion.

Secondly, the equal-sample-analysis solution is determined. It can be expressed in the form

$$S(t) = \frac{S_o}{1 + ct} \tag{21b}$$

Each analyzed component remains under scrutiny for the entire length of sample. The energy reported at each frequency is associated with the same

analysis sample as the energies reported for all other frequencies. This practical feature is very advantageous since a possible ambiguity in measurements is avoided. Appendix C develops these undesirable measurement errors which occur in conventional wave analyzers.

Thirdly, a linear speed-time relationship is developed, and can be expressed as

$$S(t) = S_o \left[1 - \frac{c\, f_o\, t}{(f_{am})_{max}}\right] \qquad (15a)$$

Equation 15a has limited theoretical use. The solution only serves to correlate the previous non-linear relationships and the conventional linear-frequency-sweep excitation.

A fundamental and useful characteristic exists for both the minimum-analysis-time solution and the equal-sample-analysis solution. The maximum deviation from linearity within the pass-band of the selective network is less than $\frac{1}{Q}$ * where

$$Q = \frac{f_f}{\Delta f}$$

f_f = the mid-band frequency of the selective network.

For all practical purposes the sweep rate is linear since the proposed analyzer will contain a high-Q selective network.

Figure 2.3 describes the family of decaying components, $f_{bn}(t)$, sweeping across the fixed pass-band. The minimum-analysis-time solution exhibits equal sweep rates at f_u for all values of $f_{bn}(t)$. Hence, if one rate is critical, all rates are equally critical. On the other hand, the equal-sample-analysis solution can have only one critical sweep rate at f_u: this occurs for the derived component $f_{bo}(t)$ associated with the lowest recorded frequency component, f_{ao}. All other sweep rates within the pass-band will oversatisfy the characteristic time requirements of the selective network.

---

* See Equations 14 and 23 in Appendix B.

The Response of the Fixed Selective Network to a Varying Frequency Excitation

Two basic multiplier relationships are presented in Equations 11b and 21b. Both expressions are non-linear functions. Yet, within the pass-band of the selective network the excitation function

$$e(t) = E \cos \Theta \approx E \cos \left(\frac{d\Theta}{dt}\right) = E \cos \left[2\pi f_{bm}(t)\right]$$

follows very nearly a linear sweep rate.

The three multiplier relationships can be expanded in a Maclaurin's Series in angular displacement, θ, of the form

$$\theta = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + \ldots$$

The first three terms for two basic multiplier relationships are identically equal to the linear speed-time expansion resulting from Equation 15a. Within the pass-band the t^3 and all subsequent terms are considered negligible.

In Appendix B the excitation of a simple LCR circuit is investigated. The response to a nearly linear sweep excitation function is evaluated by the real convolution integral

$$r(t) = \int_{-\infty}^{t} e(\tau)\, h(t - \tau)\, d\tau \tag{25}$$

where

$r(t)$ = the response of the system.

$e(t)$ = the excitation function of the system.

$h(t)$ = the unit impulse response of the linear system.

This integral is broken down analytically using the method of Barber and Ursell.[2] The derivation is included in the thesis since: (1) the solution provides a better understanding of the basic parameters involved, (2) the derivation conveys a physical picture of the response and (3) this approach provides a logical starting point for future studies concerned with the effects of ignoring the t^3 and any subsequent terms in the expansion of θ. Barber and Ursell studied a mechanical analogy to our

system, which is a variable-speed optical analyzer employing a resonant vibration galvanometer. Therefore, it is not surprising that the eventual solutions contained in Equations 34 and 36 are of the same form as the results obtained by Barber and Ursell. These authors have plotted envelopes of this transient resonance.[2,3] Their results, with appropriate changes in notation, are presented in Figure 2.4. Similar results would be obtained by use of any linear, second-order system. Universal resonance curves are made available which can be applied to a very good approximation. These are useful since many complex selective networks can be approximated by a linear second-order system. For example, Scanes[12] describes a parallel-T network which is exactly equivalent to a series LCR circuit.

Curve a of Figure 2.4 is the characteristic steady-state response with the peak arbitrarily set at zero decibels. For curves b through g the amplitudes of the peak input signal would be underestimated by the indicated power error. A simple relationship exists between this dynamic depression error and the parameter K.[*] Also note that the peak transmission always occurs after the instant when the excitation frequency equal f_r. This frequency displacement distortion is also related to the parameter K.[**] Study of Figure 2.4 indicates that the effective band-width, $(\Delta f)_e$, is greater than the nominal band-width, Δf.[***] Furthermore, secondary peaks are observed which could mask the detection of weak input components.

Note that Figure 2.4 does not take into account two important aspects of the general problem which should be considered for the proposed analyzer.

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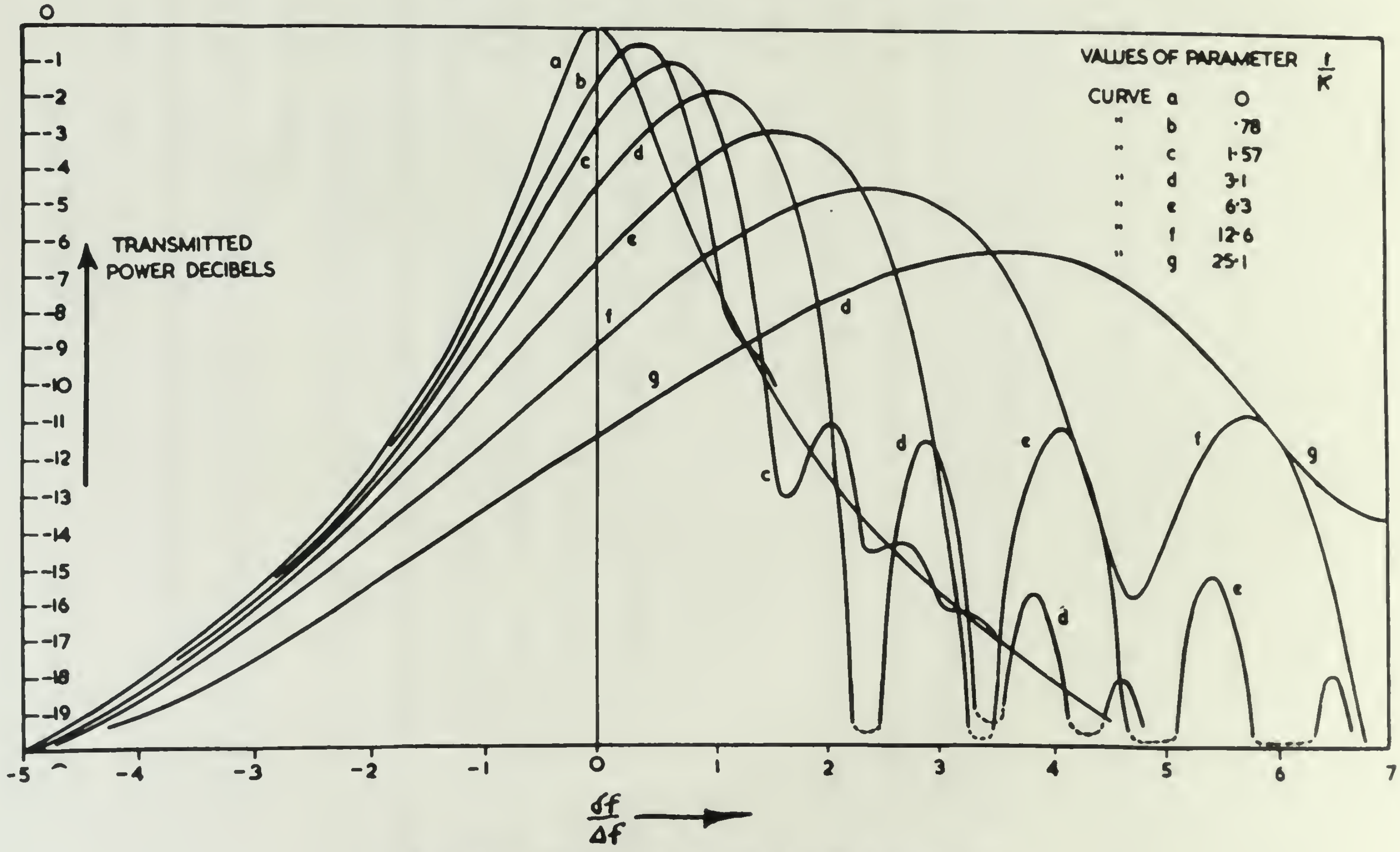
[*] See Figure 3.3

[*] See Figure 3.4   based on several sources including Figure 2.4

[*] See Figure 3.5

FIGURE 2.4
TRANSMISSION OF A SIMPLE SELECTIVE
NETWORK IN CONTINUOUS FREQUENCY ANALYSIS



[Source: Reference 3 and Thesis Equation 36]

The first occurs when initial conditions are not zero. Marique[18] and Soanes[19,20] each discuss the additional transient term which exists in the response. The second aspect is the determination of the effect of starting the sweep in or near the pass-band. Soanes[19] presents a solution to this problem by an extension of the work of Barber and Ursell. Instead of building up to the maximum response in a smooth fashion as indicated in Figure 2.4, "oscillations" occur near the beginning of the sweep.

In conclusion, six different distortions can exist:

(1) amplitude depression, (2) frequency displacement, (3) decreased selectivity, Q, (4) secondary peak masking (ringing), (5) initial-conditions transients and (6) uneven leading edges resulting from starting the sweep in or near the pass-band.

Magnetic Recording Characteristics

Modern tape is sufficiently constant in sensitivity to allow amplitude recording to a few percent.[24] It has been shown that there exist workable analyzers which employ a magnetic medium. Hence, magnetic properties and performance characteristics of recording tapes do not in themselves determine the feasibility of the proposed analyzer. Yet, at least three of these characteristics must be recognized as major factors in the overall design.

First, there is a distinct advantage in continuous operation at a reproduce speed which is always greater than the constant record speed. Use of multiplication rather than division results in increased output magnitudes and increased signal-to-noise ratios. However, in order to keep the number of selective networks and graphic recorder channels to a minimum it may be necessary to partially speed-up and partially slow-

down (as compared to the fixed recorded speed) the reproduce tape for a wide frequency-band analysis. Secondly, a definite limitation occurs because of imperfect magnetic contact between the reproducing head and the recording medium. A rigid requirement for this analyzer is that it provides constant contact of the playback head against the magnetic tape. R. L. Wallace, Jr. of the Bell Laboratories[31] has made experimental and theoretical determinations of the spacing losses involved when the reproduce head loses contact with the magnetic surface. He reports that

$$\text{Spacing loss (db)} = 55\,(d/\lambda)$$

where

d = spacing introduced between reproducing head and magnetic medium

λ = recorded wavelength = $\frac{S_r}{f_{an}}$

db = decibels that reproduced votage level is decreased

It is to be expected that the magnetic contact between the reproduce head and the medium is less than perfect. Modulation noise introduces a spacing loss for devices exhibiting only apparent intimate contact. For example, imperfect magnetic contact can result from chattering of the tape on the reproduce head, or from changes in the degree of contact due to clumps on the tape surface.[23] Amplitude modulation of the reproduced signal results. Specifically, let us consider the case when a non-magnetic disc with tape wrapped around the circumference is used as the multiplier. Any eccentricity in the motion of the disc will introduce the amplitude modulation described above. Furthermore, this eccentricity will result in excessive wear in the heads, which increases the area in contact with the tape and decreases the signal-to-noise ratio.

Lastly, a characteristic high frequency loss due to the length of the playback head gap will probably determine the upper frequency limit

for the proposed analyzer. When the reproduce wave length ($\lambda = \frac{S_r}{f_{ao}}$) approaches the effective playback gap length, β, a decreased signal output will occur. Lennert[27] predicts that this loss can be expressed as

$$\text{Voltage level loss} = 20 \log_{10} \frac{\sin\frac{\pi\beta}{\lambda}}{\frac{\pi\beta}{\lambda}}$$

Hence, there exists a certain minimum wave length the reproduce head will recognize.

Sample Length Considerations

Appendix D consists of an investigation on sample length requirements. The results can be summarized as

$$L \text{ (seconds)} = \frac{2\,K\,f_u}{f_{ao}\Delta f} \tag{39}$$

$$L \text{ (cycles)} = \frac{2\,K\,f_u}{\Delta f} \tag{39a}$$

where

L = recorded sample length

Summary

Six key questions regarding the proposed analyzer were raised at the end of Chapter I. A basic theory has been formulated in Chapter II-A, and in Appendices B, C, and D which provides a basis for answering these questions.

CHAPTER II-B

EXPERIMENTAL PROCEDURE

Introduction

Figure 1.1 showed the functional operations necessary to be performed on an unknown recorded input in order to extract the desired spectral density of that input. Figure 2.5 shows in block diagram form the arrangement of components which was used to accomplish the aforementioned functions. The following discussion of these components refers to those actually used. A more thorough discussion of major components to accomplish the desired functions is given in Appendix E.

Outline of Operation of Block Diagram

The unknown sample to be analyzed is taken as the starting point for the block diagram. This tape is wrapped on the periphery of circular disc 9.00 inches in diameter. A reproduce head is mounted so that the magnetic tape on the disc revolves past the face of the reproduce head. The movement of the tape past the gap of the reproduce head causes a voltage to be generated in the windings of the head in accordance with the signal on the tape as modified by the linear speed of transition of the tape past the gap. Because the voltage output of the reproduce head is rather low (0.004 volts maximum for the heads used, provided that the tape is in contact with the face of the head) a voltage amplifier is used to raise the level of this out-put. The output of the amplifier may be composed of many different frequencies depending on the character of the recorded input. The selective system picks out one frequency component so that this component can be measured. The selective system for this arrangement is a high Q parallel resonant tuned circuit employing positive feedback for Q multiplication. The output of the tuned circuit is fed through

a cathode follower isolation stage to a conventional linear detector. The detector determines the amplitude of the single frequency output of the tuned circuit as a function of time, and this amplitude, varying at a much slower rate than the undetected output of the resonant circuit, is capable of being recorded permanently on a low-frequency, paper recorder.

The speed of the disc carrying the magnetic tape is controlled by a Ward-Leonard type of speed control. (However, this control system was run open loop using the typical speed characteristics of an armature controlled, direct current motor rather than closed loop with tachometer feedback as originally planned) The controlling signal to the Ward-Leonard system is developed in a function generator. Only an exponential variation was investigated and this voltage was developed in a simple RC exponential decay circuit. The DC output of the function generator is amplified by the voltage amplifier and the power amplifier controls the field current of a DC generator whose armature output is connected in series with the armature of a DC motor. The field of this motor is supplied from an external constant source. Thus the combined system provides an armature controlled DC motor with which the speed of the disc might be controlled. The controlling voltage and the speed of the motor, indicated by the output voltage of a tachometer, are both monitored by paper recorders. The input to the frequency selective tuned circuit is monitored by a cathode ray oscilloscope.

It was determined that the driving arrangement for this system resulted in an uneven variation of frequency as the played-back frequency of a recorded component was swept across the tuned circuit. In order to achieve a smooth variation of excitation frequency for the tuned circuit, the disc was gotten up to the desired speed and then was permitted to slow down by

itself. The butt joint transient of the recorded sample was used as a marker in order to indicate the variation in speed of the free running disc. This transient was recorded along with the response of the tuned circuit on a twin paper recorder. The revolutions of the disc as indicated by the butt joint markers were calibrated in time by using one second markers which were imposed on a third trace of the paper recorder. A block diagram of this arrangement is given in Figure 2.6.

FIGURE 2.5
Block Diagram Driven System

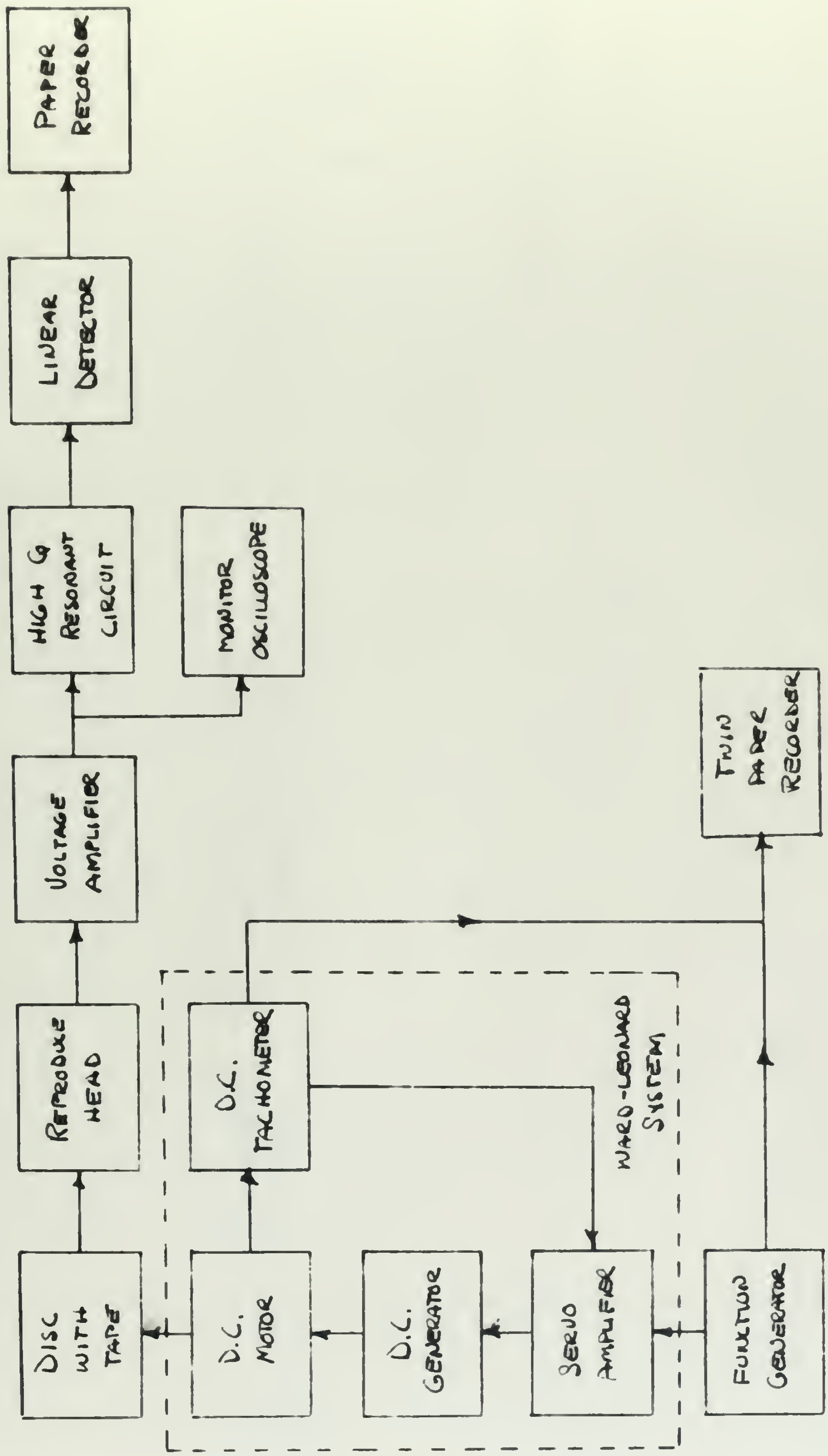
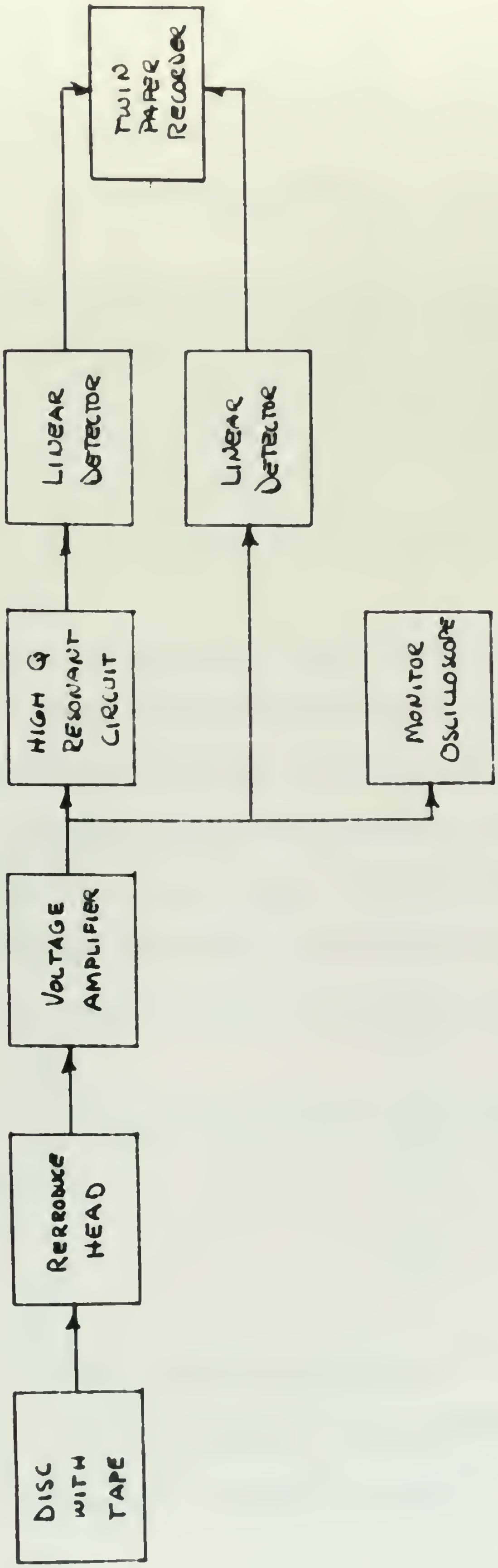

PAPER RECORDER
LINEAR DETECTOR
HIGH Q RESONANT CIRCUIT
MONITOR OSCILLOSCOPE
VOLTAGE AMPLIFIER
REPRODUCE HEAD
TWIN PAPER RECORDER
D.C. TACHOMETER
WARD-LEONARD SYSTEM
DISC WITH TAPE
D.C. MOTOR
D.C. GENERATOR
SERVO AMPLIFIER
FUNCTION GENERATOR

FIGURE 2.6

Block Diagram - Free Running System


DISC WITH TAPE
REPRODUCE HEAD
VOLTAGE AMPLIFIER
HIGH Q RESONANT CIRCUIT
MONITOR OSCILLOSCOPE
LINEAR DETECTOR
LINEAR DETECTOR
TWIN PAPER RECORDER

CHAPTER III

RESULTS

Theoretical Results

A basic analyzer theory has been formulated in Chapter II-A, and in Appendices B, C, and D. The following results of this investigation are of specific interest:

(1) Two multiplier speed-time relationships were determined and are presented in Figures 3.1 and 3.2. For the minimum-analysis-time solution each analyzed component remains within the pass-band of the selective network the minimum time for acceptable analysis. For the equal-sample-analysis solution each analyzed component remains under scrutiny for the entire length of sample.

(2) The term acceptable analysis is based on having the output of the selective network remain within tolerable limits of amplitude and frequency distortion which are imposed by the characteristics of the selective network. Figures 3.3, 3.4, and 3.5 predict this distortion as a function of a parameter K. This constant relates the characteristics of the excitation frequency with those of the selective network. Note that these results are applicable only for the range of K indicated in each figure.

(3) The recorded sample of tape selected for analysis must have a certain minimum length

$$L = \frac{2 K f_u}{f_{ao} \Delta f}$$

where

L = length of sample measured in seconds.

f_u = the upper cut-off frequency of the selective network.

Δf = the band-width of the selective network.

f_{ao} = the lowest recorded frequency component being analyzed.

$$K = \frac{(\Delta f)^2}{\frac{df_{bn}}{dt}} \text{ cycles}$$

f_{bn} = the output of the frequency multiplier

Experimental Results

Two factors have combined to make the experimental results largely qualitative rather than quantitative. These are

(1) Variation of the amplitude of the reproduced signal with angular position of the disc due to the eccentricity of the disc.

(2) Inability to control the speed of the disc within narrow limits

Figure 3.6 shows in a qualitative manner the analysis of a composite recorded sample. The recorded frequencies and the analyzed frequencies are given below. It is important to note that the recorded sample has not been analyzed by an independent means.

Recording Frequencies		Analyzed Frequencies	
Frequency	Relative Amplitude	Frequency	Relative Amplitude
		184	.71
		188	1.56
198	1.00	198	1.00
393	1.00	392	.71
		408	.36
418	1.00	418	.93
		618	4.42
686	.50		
696	1.00	694	.71
		842	.20
886	1.00		
891	.50		
1180	1.00		

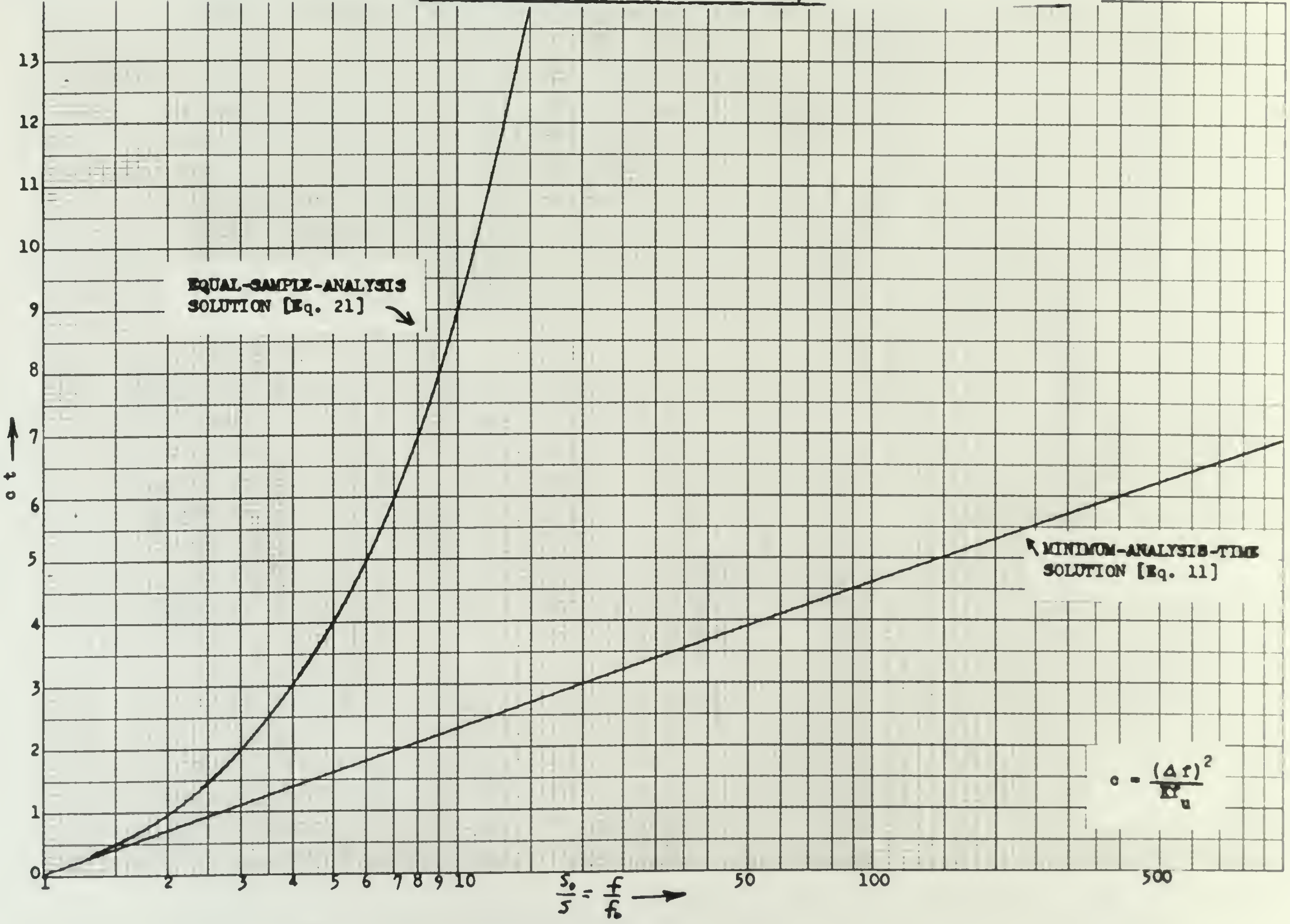
The disc for the above analysis was permitted to slow down freely. The variation of the linear speed of the tape is given in figure 3.8.

Figure 3.7 gives a measure of the resolution of the analyzer for a Q of the tuned circuit of 625, and a resonant frequency of 5020. The speed-time relationship is similar to that given in Figure 3.8 and results in a parameter of K equal to 2.46 at 400 cycles per second and a value of K equal to 1.47 at 700 cycles per second.

The shape of the output pulse for one recorded frequency, 300 cycles per second is given in Figure 3.9 for different values of the parameter K.

FIGURE 3.1

MULTIPLIER SPEED-TIME RELATIONSHIPS, I



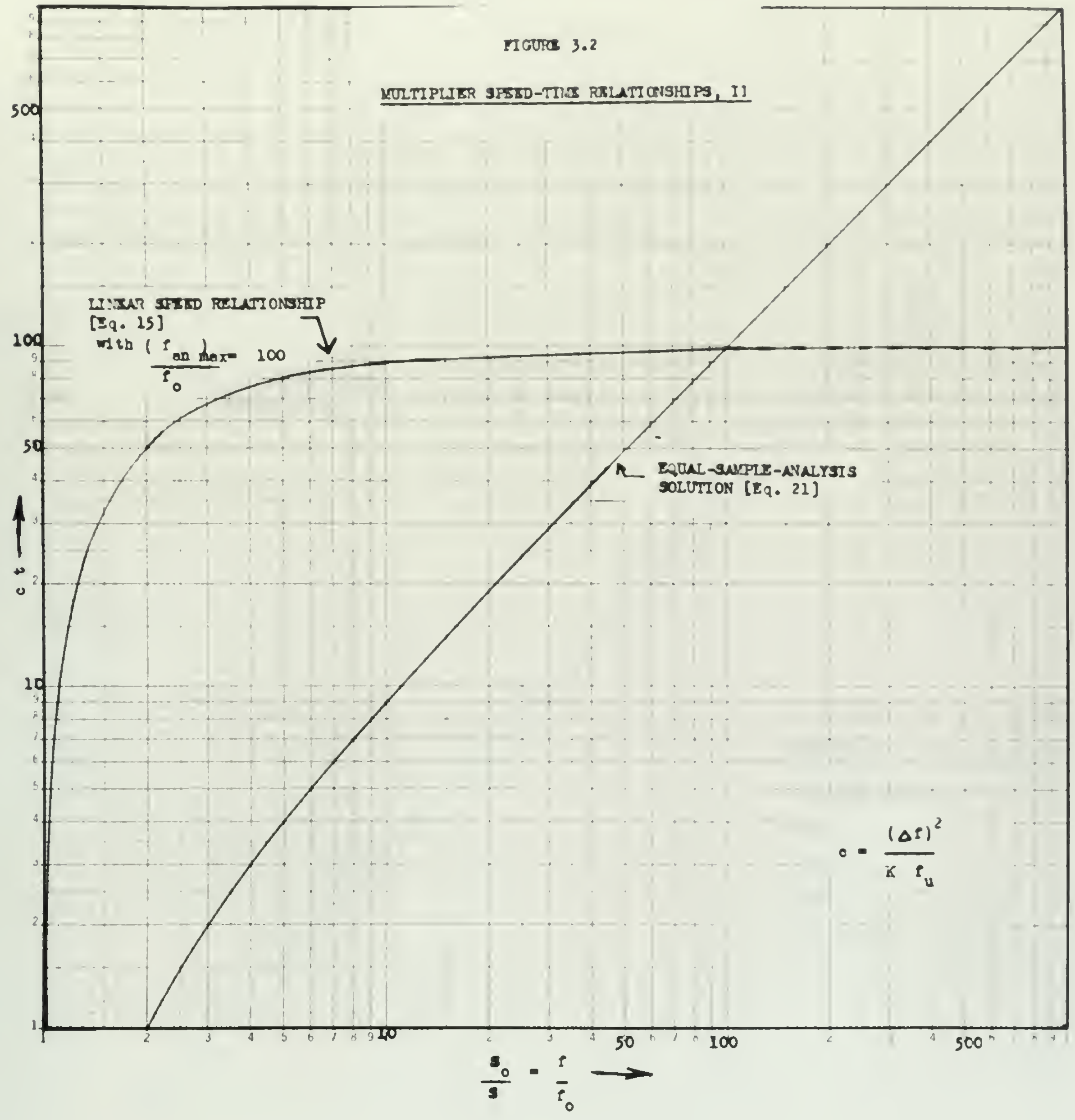


FIGURE 3.2

MULTIPLIER SPEED-TIME RELATIONSHIPS, II

FIGURE 3.3

PEAK TRANSMISSION POWER ERROR FOR

CONTINUOUS ANALYSIS OF SIMPLE SELECTIVE NETWORK

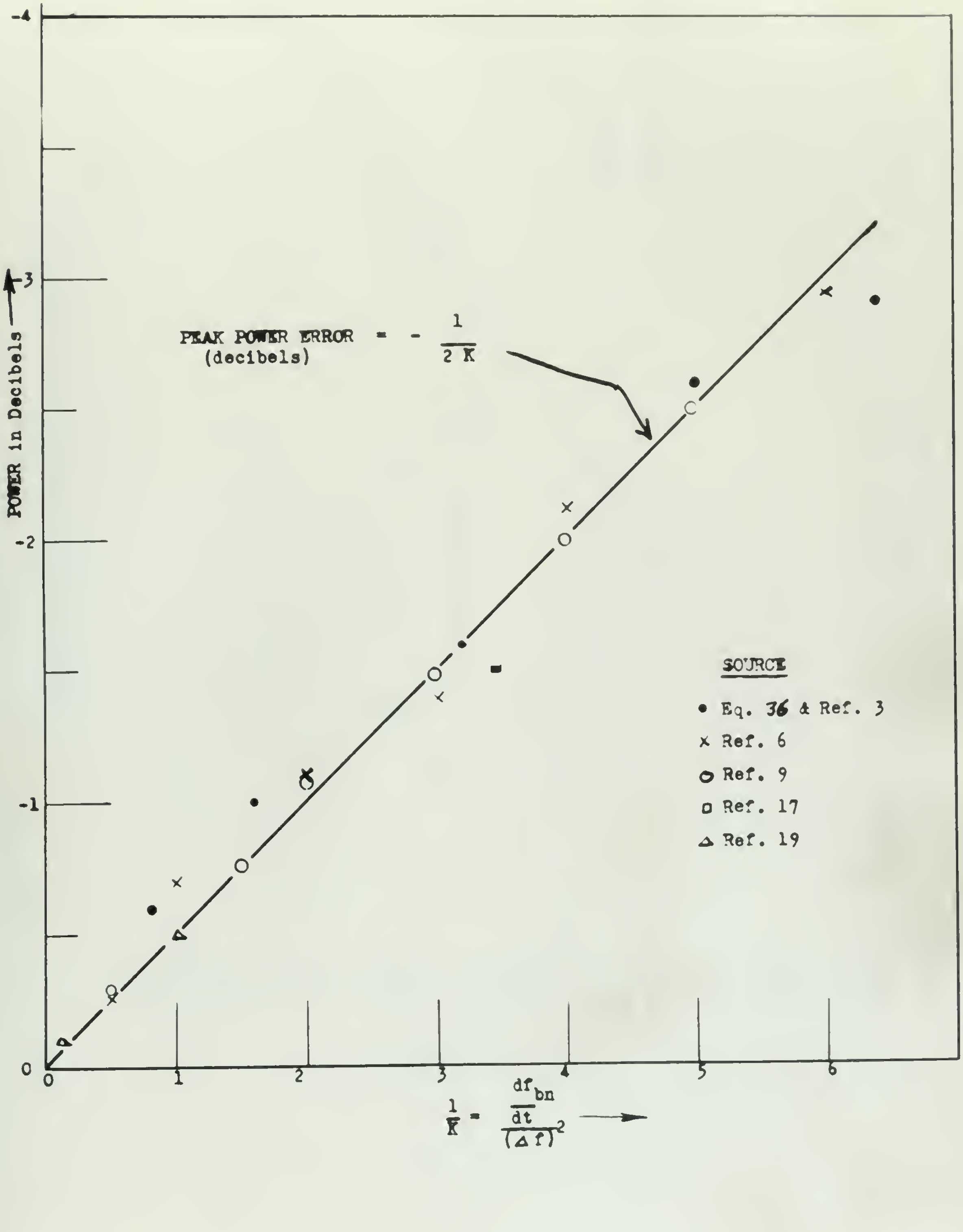


FIGURE 3.4

FREQUENCY LAG IN PEAK TRANSMISSION FOR CONTINUOUS ANALYSIS

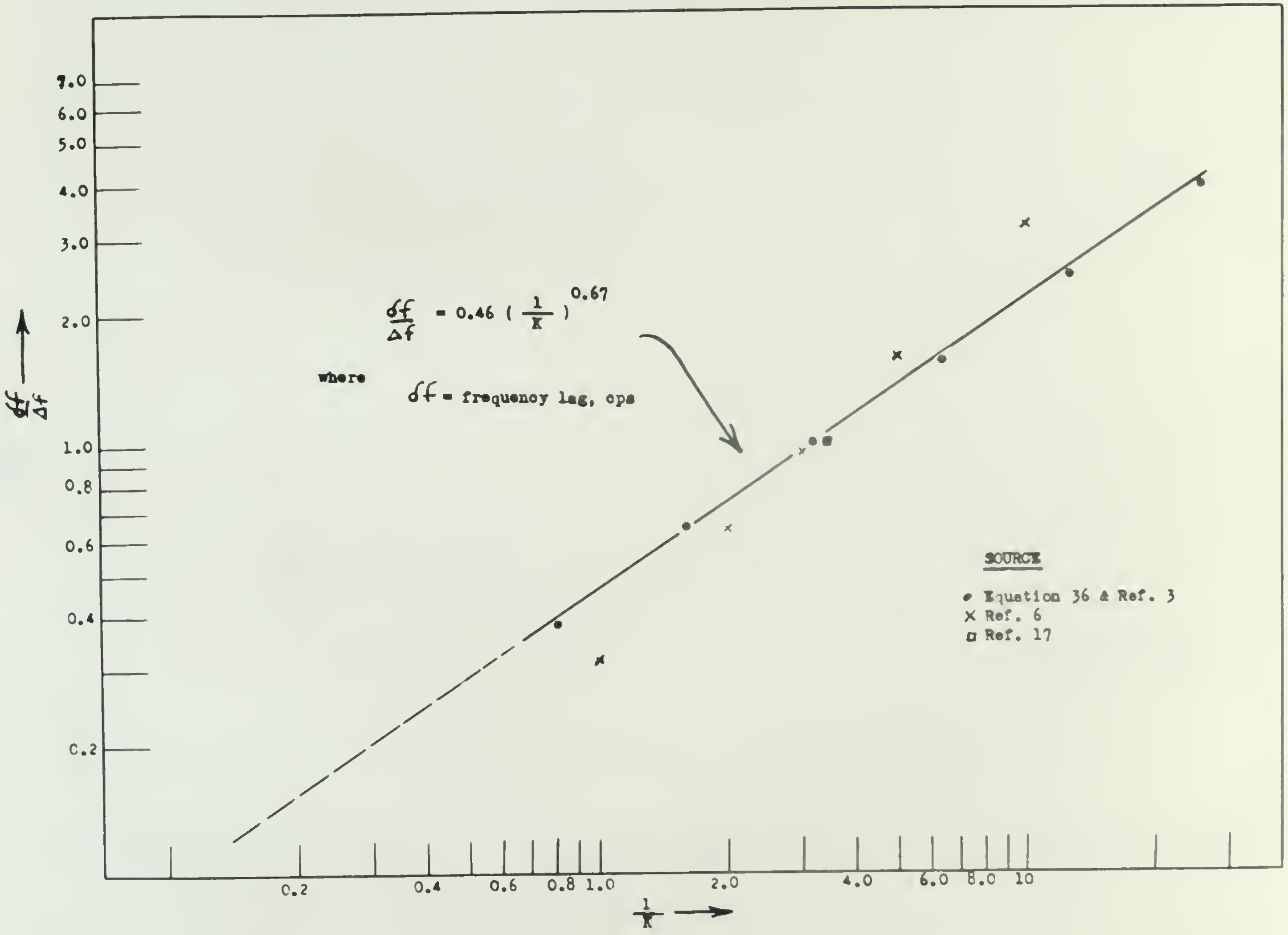


FIGURE 3.5

RELATIONSHIP BETWEEN EFFECTIVE BAND-WIDTH, $(\Delta f)_e$, AND NOMINAL BAND-WIDTH, (Δf), FOR CONTINUOUS ANALYSIS

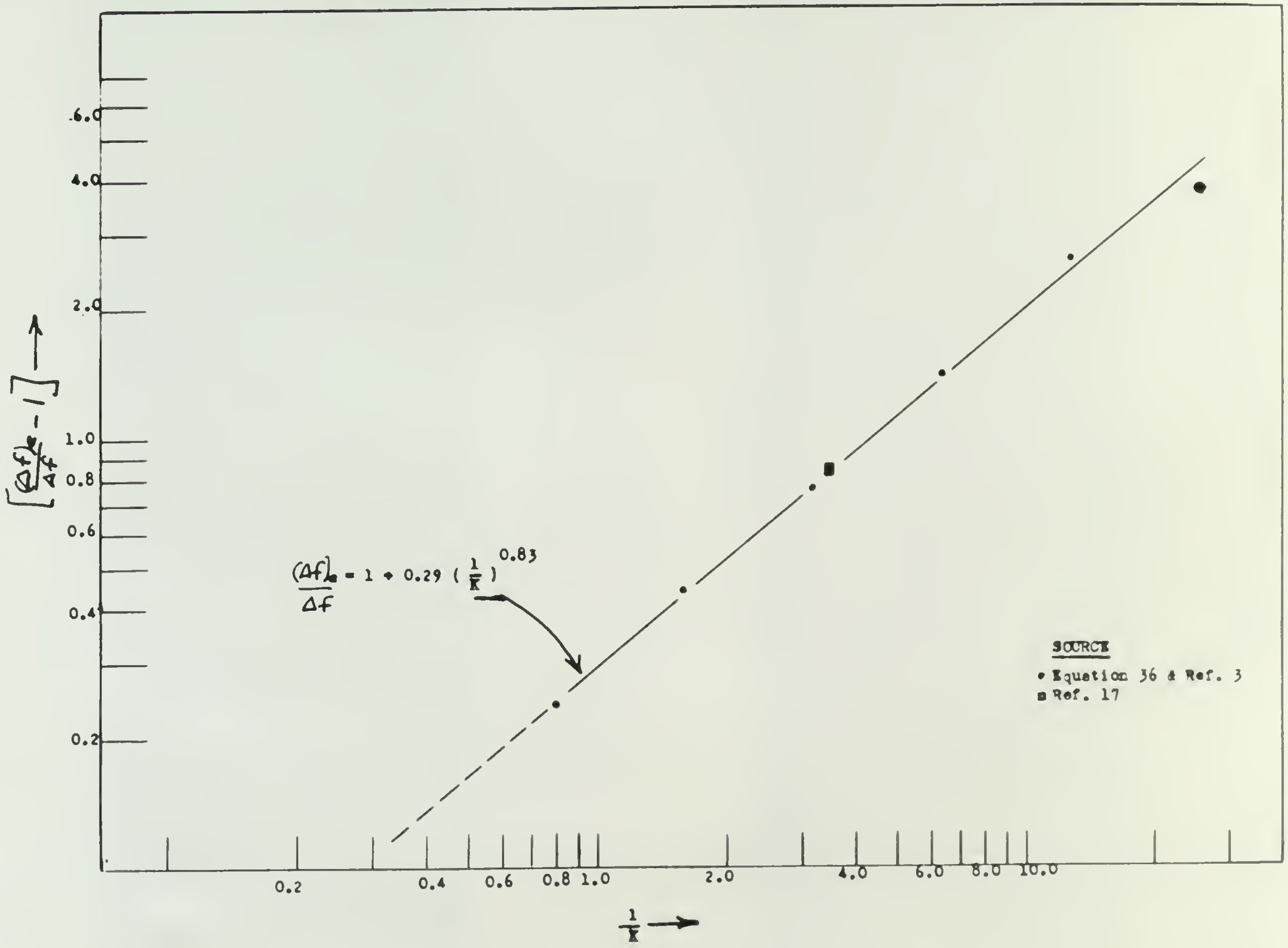


FIGURE 3.6

Analyzer Indication Of Recorded Components

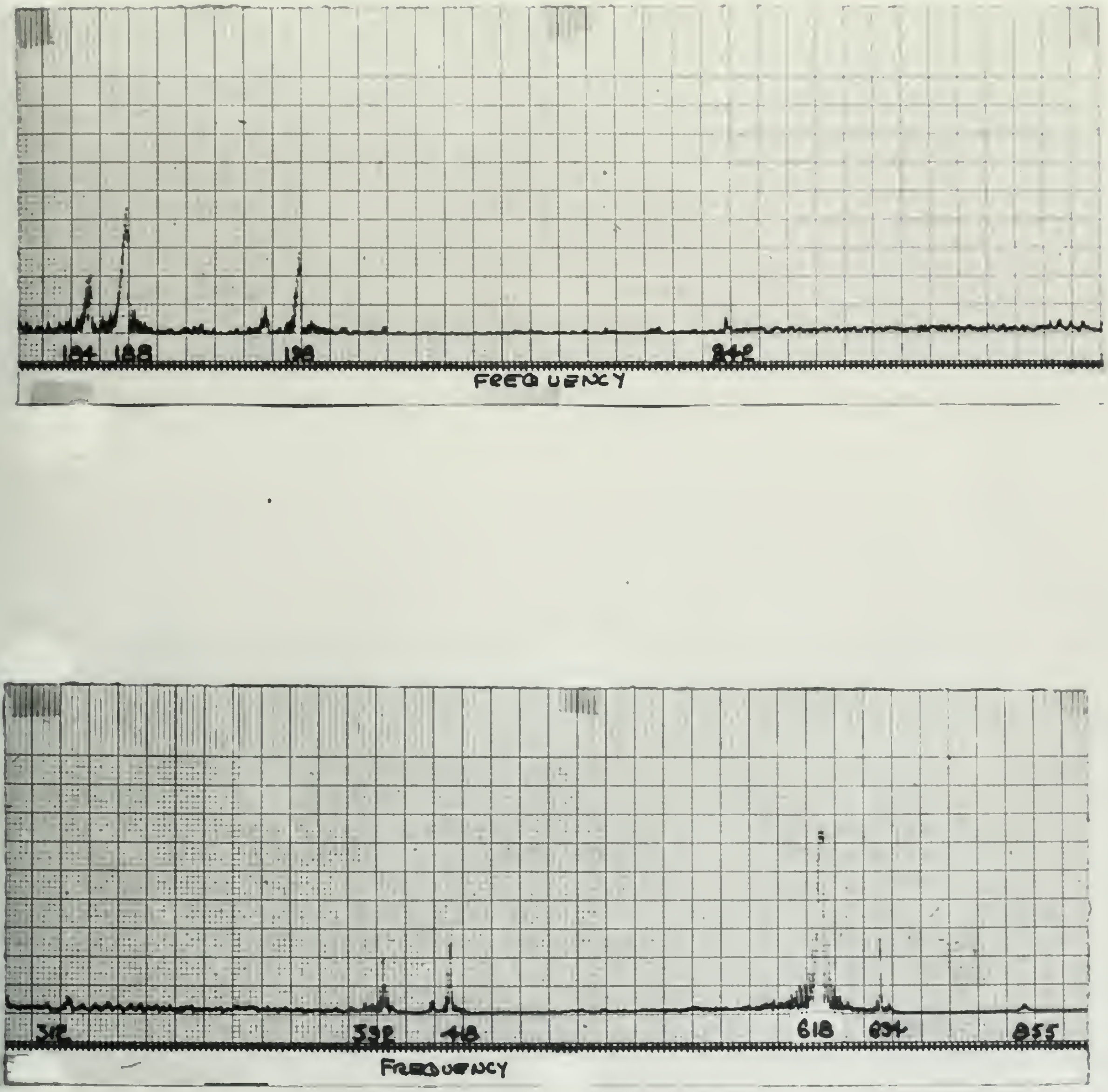


FIGURE 3.7

Expanded Indication of Two Recorded Components Showing Resolution of Analyzer

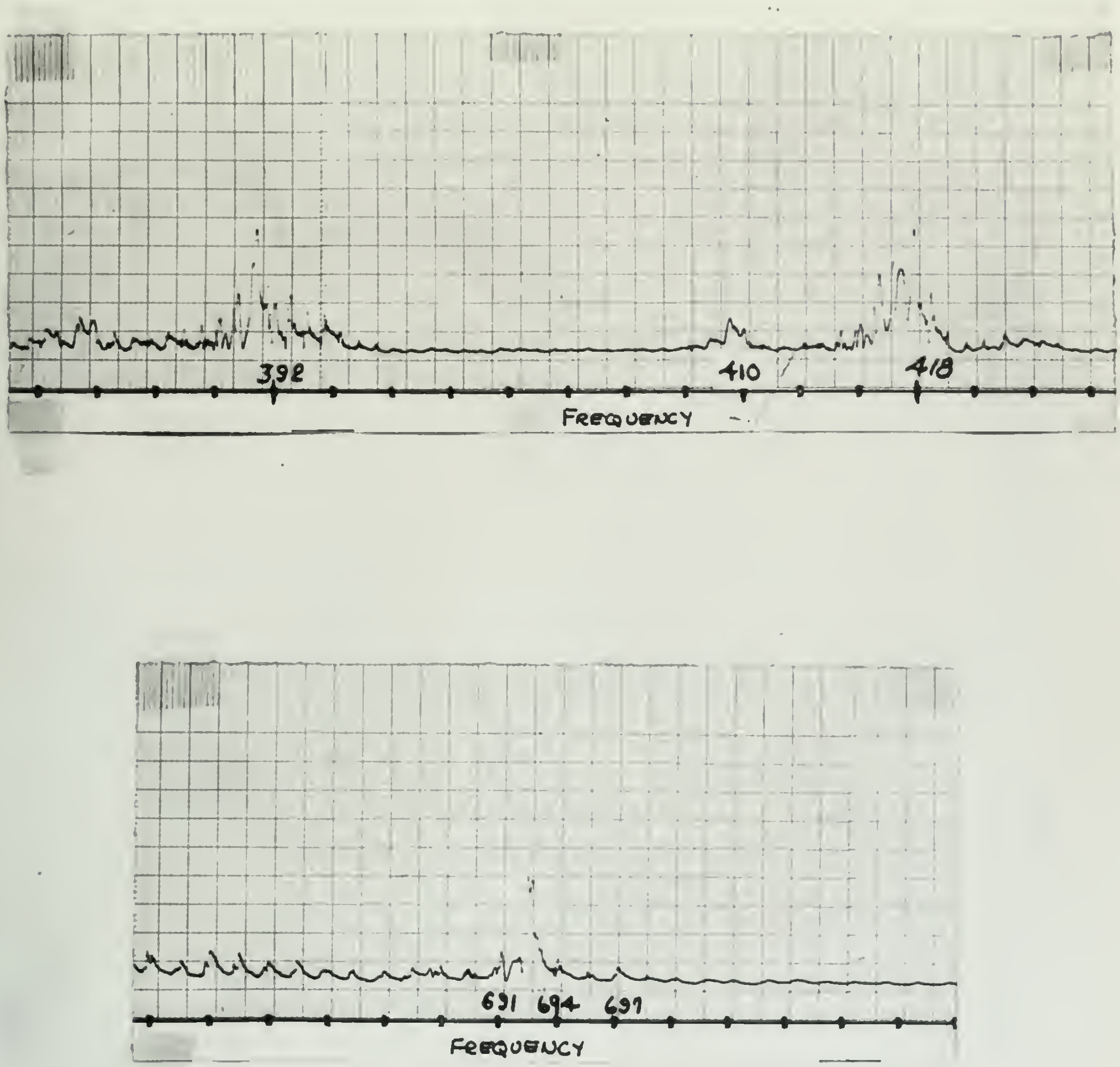


FIGURE 3.8

Free Running Speed Time Relationship

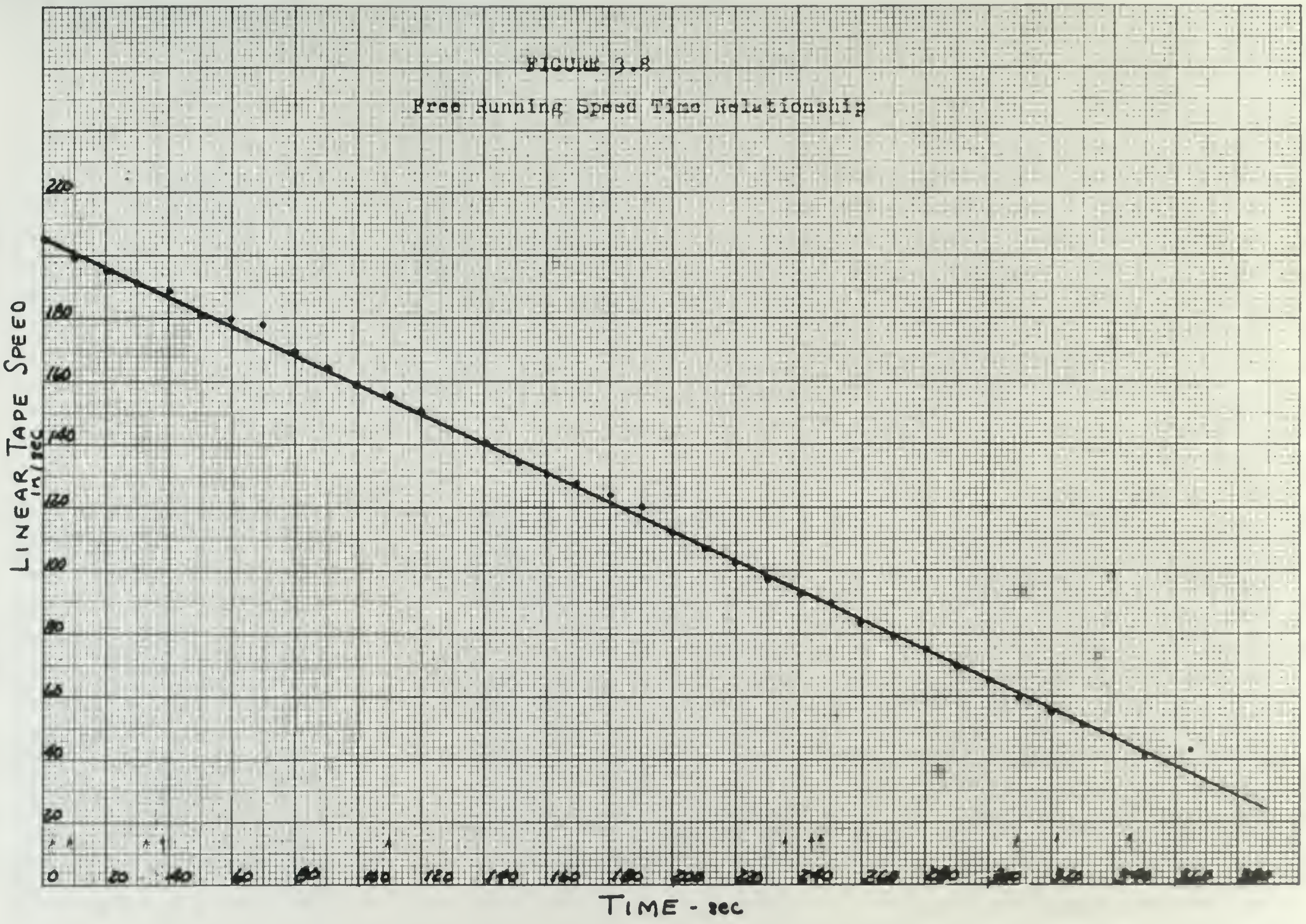
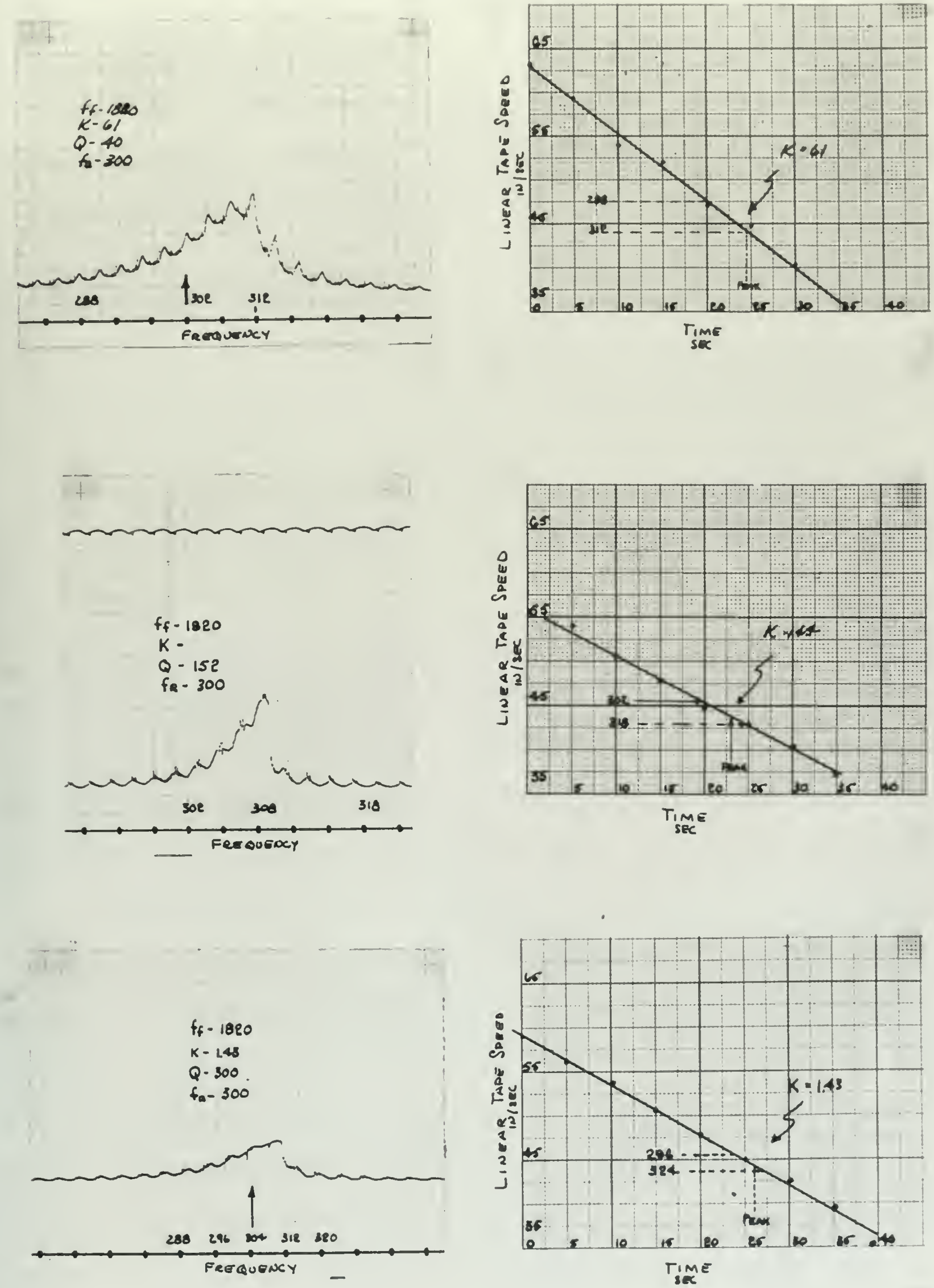


FIGURE 5.7

Output Pulse Shapes for Different Q's



CHAPTER IV

DISCUSSION AND CONCLUSIONS

Discussion of Theoretical Results

A basic analyzer theory has been formulated in Chapter II-A, and in Appendices B, C, and D. In general, the Theory provides a basis for answering the questions raised at the end of Chapter I.

Which Speed-Time Relationship?

Two basic relationships have been determined: the minimum-analysis-time solution and the equal-sample analysis solution. The equations were derived by a qualitative approach. Fundamentally these are based on the fact that the excitation frequency sweep is very nearly linear while within the pass-band of the high-Q selective network. The deviation from linearity is less than 1/Q.

A basic criterion for any automatic analyzer is to complete the analysis in the minimum possible time compatible with the desired analyzer characteristics. This basic need prompted an investigation into the response of a simple selective network to a nearly linear sweep excitation function. Figures 3.3 to 3.5 correlate the results of Appendix B with other theoretical studies currently published in the literature. Note that these curves are only good for the range of K indicated. For this analyzer a certain minimum value of K is considered essential in order that secondary masking does not predominate. Barber[8] concentrates on wider ranges in K and presents correction values for the various distortions up to K's less than 1/25. Our plots are presented in order to provide a basis for establishing tolerable limits of distortion. It is recommended that K be equal to or greater than 1.0. This should result in peak power errors less than 1.0 db for most selective

networks which would be used in an analyzer of this type.

Significantly, one of the investigators[17] investigated the response of the network indicated in Figure B.3 to a logarithmic frequency excitation. (All other references are concerned with linear-frequency-sweep excitation.) Penfield's circuit only had a Q of 5, for which there exists a maximum deviation from linearity of 20%. Nevertheless, band-width and frequency lag distortions are exactly predictable by the equations indicated in Figures 3.4 and 3.5, while the peak power error is only 0.2 db from its predicted value. (See Figure 3.3)

Which multiplier relationship should be used in the designed analyzer? Equal sample analysis is a very desirable characteristic. This would avoid the possible time-distribution ambiguity in measurements discussed in Appendix C. Equal sample analysis is considered to be an essential characteristic for the proposed analyzer. Currently available wave analyzers do not achieve it and this would be a big selling-point in favor of this constant-percentage resolution device. Yet, it should be noted from Figures 3.1 and 3.2 that equal sample analysis sometimes requires excessive analysis time as compared with the minimum-analysis-time solution. Another solution would be to increase the number of frequency bands analyzed. However, increased numbers of filters and recorder channels tends to over-complicate what is basically a very simple device. It is anticipated that the wave analyzer could quite possibly consist of a multiplier speed-time relationship made up in part by segments based on both equal-sample-analysis and minimum-analysis-time requirements.

Technically the equal-sample-analysis solution does not provide constant percentage resolution since K does not stay constant. Figure 3.5 indicates that effective band-width becomes smaller, and percentage resolution improves as K increases. Yet for all practical purposes schemes which would be used in an analyzer of this type.

Significantly, one of the investigators[11] investigated the response of the network indicated in Figure 2.3 to a logarithmic frequency excitation. (All other references are concerned with linear-frequency-sweep excitation.) Penfield's circuit only had a Q of 5, for which there exists a maximum deviation from linearity of 6%. Nevertheless, band-width and frequency lag distortions are exactly predictable by the equations indicated in Figures 3.4 and 3.5 while the peak power error is only 0.8 db from the predicted value. (See Figure 3.9)

Which multiplier relationship should be used in the designed analyzer? Equal sample analysis is a very desirable characteristic. This would avoid the possible time-distribution ambiguity in measurements discussed in Appendix C. Equal sample analysis is considered to be an essential characteristic for the proposed analyzer. Currently available wave analyzers do not achieve it and this would be a big selling-point in favor of this constant-percentage resolution device. Yet, it should be noted from Figures 3.1 and 3.2 that equal sample analysis condition requires excessive analysis time as compared with the minimum-analysis-time solution. Another solution would be to increase the number of frequency bands analyzed. However, increased numbers of filters and recorder channels leads to over-complexity what is basically a very simple device. It is anticipated that the wave analyzer could quite possibly consist of a multiplier speed-time relationship made up in part by segments based on both equal-sample-analysis and minimum-analysis-time requirements.

Technically the equal-sample-analysis solution does not provide constant percentage resolution since K does not stay constant. Figure 3.5 indicates that effective band-width becomes smaller, and percentage resolution improves as K increases. Yet for all practical purposes constant-percentage resolution results. For a high-Q network the difference in resolution for a frequency band is small.

The Influence of N(t)

The speed-time relationship directly affects the averaging and indicating devices. This is clearly indicated in Figure 1.1.

Consider the indicator. The graphic recorder must know what frequency is being analysed at any instant. It is desirable that: (1) the recorder tape have equal intervals of frequency divisions, and (2) this tape be a pre-printed affair. For the minimum-analysis-time solution the recorder paper can move at a constant speed. An amplitude versus log f plot results. However, for the desired equal-sample-analysis solution the paper will not run at a constant speed, but instead will follow its own speed-time relationship.

This complication extends to the averaging device. For example in equal sample analysis the 600 cps component remains within the averaging device three times as long as the 200 cps component. The necessary design requirements for the averaging device must be investigated, as must the basic philosophy underlying the averaging circuits for this analyzer.

Discussion of Experimental Results

The tape for the one run reported for different frequency components has not been analyzed as yet by independent means. It is obvious that there is more on the tape than was supposedly recorded. An oscilloscope was set up to monitor the output of the selective circuit before detection in order to insure that these extra signals were actually being imposed on the selective system. This presentation corroborated the paper recording. The presence of these signals can be explained by inoperative erasing during the recording of the signals. The tape for

this analysis was prepared by simultaneous recording of the outputs of several audio oscillators and only the outputs were measured, not the components as actually recorded on the tape. The important thing for this sample is not that extra signals are on the tape but the closeness, with which the several frequencies as recorded were actually analyzed.

During the progress of this thesis it was decided that an amplitude comparison could not be made for the various analyzed frequencies due to (1) the pulse variation due to the eccentricity of the shaft whereby the null of the pulse might add or subtract to the maximum response of a constant amplitude signal, and (2) insertion loss due to the air gap. Nevertheless it is interesting to note the rough correlation between the relative magnitudes of the recording oscillators and those frequency components which did show up on the tape.

A Q of 625 would give a bandwidth of about 0.3 cycles at 400 cycles. Clearly the resolution which was obtained is not this good. It is an important fact that the Q of the circuit based only on the half power points is a poor indication of the response of the system. Another important feature of a resonant circuit is the nature of the resonant curve outside the bandpass. These skirts could easily lead to the poorer resolution as obtained with this analyzer.

Actually such a high value of Q would not be necessary for wave analysis since a resolution of 1% is normally acceptable. The Q in this case was set high because it was desired to have a K parameter value of about 1. With the free running case it was necessary to set the Q up. Due to the difficulty in reading the General Radio frequency meter dial at the resonant frequency of the tuned circuit, it is estimated that this Q could be in error approaching 100%. However, this

value of Q would not explain the poorer resolution of the analyzer as would be expected by a resonant system specified only by its Q. From the expanded indication of Figure 3.7 which was obtained on the fast speed of the recorder, it can be seen that another frequency component separated by two cycles would be resolved. This would correspond to a resolution of approximately 0.3%. The absence of the frequency component at 686 cycles per second can only be explained by the fact that the component was not actually recorded and not that it was obscured by the frequency at 696 cycles.

The higher frequencies which were supposedly recorded on the tape would not show up to any large extent. This fact is due to the rapid change of disc speed at the analysis of these higher frequencies (or low disc speed) This fact is not immediately obvious from the speed time relationship of Figure 3.8 but could be seen by the rapid variation of the butt joint markers in this range. Consequently the amplitude of the reproduced signal would be attenuated due to (1) a parameter value of K less than 1.00, (2) an increase in the insertion loss at higher recorded frequencies and (3) the normal low frequency drop off of a recorded signal when the recorded wavelength approaches and exceeds the dimensions of the reproduce head. Without belaboring these results any more, it is felt that an accurate analysis using this device could be made from the view point of frequency alone and under certain conditions the amplitude of various frequency components could be measured within tolerable limits.

Conclusions

The experimental investigation of the analysis brought out several real and important drawbacks in the equipment which would be necessary for this analysis. The first is that it is necessary to maintain con-

tact between the reproduce head and the tape and at the very least, any air gap should remain constant for the entire length of sample which is being analyzed. Any air gap is accompanied with a considerable equalization problem if it is desired to analyze any moderate range of frequencies. The second essential disadvantage of the analyzer is the necessity to achieve accurate speed control of the disc or tape. With these inherent drawbacks in mind a reassessment of the entire wave analysis problem was made in order to determine if some concrete conclusions might be made. In particular the features of this analyzer were compared to a conventional heterodyne type analyzer for the desirable characteristics of an analyzer. The results of this reassessment are given below:

Characteristic	Heterodyne	Variable Speed Tape
Maximum resolution	Easily accomplished	Easily accomplished
Constant percentage resolution	Would require some device which would vary the Q of the resonant system as the resonant frequency were varied	Inherent
Automatic operation	Can be accomplished	Inherent
Minimum analysis time with automatic operation	Would require multiplication of frequency of samples to take advantage of faster analysis possible at higher resonant frequency	Inherent in that multiplication of frequencies is inherent
Ambiguity problem	Easily accomplished by magnetic tape storage	Can be made inherent

From these results it is seen that the variable speed magnetic tape analyzer inherently embodies many of the desired characteristics. However, not all these characteristics are desired at the same time and the results must be considered from the viewpoint of one or two specific characteristics

for some applications. Nevertheless, the feature of using magnetic tape and multiplying frequencies in itself makes many of these other characteristics easily obtainable. A corresponding characteristic for the heterodyne would require a much more elaborate device than is used now. Consequently, it is concluded that any more elaborate equipment could just as well be put into making the variable speed magnetic tape work properly. In truth the equipment for the variable magnetic speed analyzer need not be more elaborate than that which was used in this investigation; it need only be more accurate or function more accurately.

A summary of conclusions is given below:

(1) Frequency measurements with good accuracy and with good resolution are possible provided that the frequency components of the sample to be analyzed are of the same order of magnitude.

(2) Measurement of the amplitudes of various frequency components accurately can be made provided the problems of air gap variation and tape speed variation can be solved by more precise experimentation or by different equipment.

(3) Equal sample time analysis would require a detailed investigation of possible averaging devices and indicators to display the information.

(4) The butt joint is not a factor in the analyzer provided that there is a sufficient length of sample.

Recommendations

(1) Further study should be made of what types of recorded samples this type of device can measure. Analysis was made on the basis of sinusoidal inputs.

(2) Investigate the possibility of using a loop of tape which would be transported across a reproduce head rather than affixing the tape to a drum or disc.

(3) Further investigation of the possibility of letting the drum or driving mechanism slow down due to its own or artificially introduced damping. This would involve determining an adequate means of presenting of the derived information.

(4) Investigation be made of a device which would permit averaging the response of the selective system when equal sample time analysis was being used. An indicator which would present this information should also be investigated.

APPENDIX

APPENDIX A

NOMENCLATURE

c	equals $\frac{(\Delta f)^2}{K f_u}$; $(\text{sec})^{-1}$
d	the spacing introduced between reproducing head and magnetic medium; inches.
e(t)	the excitation function of the selective network.
f	the recorded component being analyzed at time t; cps
f_a	recorded frequency components; cps.
f_b	reproduced frequency components, i.e. the multiplier output; cps
f_f	the mid-band frequency of the selective network; cps
f_1	the lower cut-off frequency of the selective network; cps
f_o	the value of f for t equals zero; cps
f_u	the upper cut-off frequency of the selective network; cps
h(t)	the unit impulse response of the selective network.
K	equals $\frac{(\Delta f)^2}{df_{bm}/dt}$; cycles.
L	length of recorded sample; seconds or cycles
N(.)	the multiplier speed time relationship.
n	takes on values 0, 1, 2, ...
Q	equals $\frac{f_f}{\Delta f}$
r(t)	the response of the selective network.
S(t)	the instantaneous multiplier speed; inches per second.
S_o	the multiplier speed at t equals zero; inches per second.
S_r	the constant recording speed; inches per second.
db	decibels.

Δf	the band-width of the selective network; cps. (equals $f_u - f_l$)
Δt	the increment of time any component f_{bn} must remain within the pass-band; seconds.
β	the effective playback gap length for the reproduce head; inches.
δ	the logarithmic decrement.
ϵ	the angular displacement for t equals zero; radians.
λ	recorded wavelength; inches
θ	angular displacement; radians
τ	the build-up time for an ideal band-pass selective network; seconds.
ω	angular frequency; radians per second.

Appendix B

THEORETICAL DETERMINATION OF THE MULTIPLIER SPEED-TIME RELATIONSHIP

The basic system has been described in Chapter II-A. (See Figure 2.1-2.3) It has been assumed that the system input can be represented as a series of n discrete sinusoidal components, each of frequency f_{an}*. The output of the multiplier is then a series of frequency modulated components, $f_{bn}(t)$. The multiplier function can be expressed in terms of the instantaneous reproduce speed, $S(t)$, and the fixed recording speed, S_R.

$$N(t) = \frac{S(t)}{S_R} \qquad (1)$$

A fundamental multiplier relationship exists:

$$f_{bn}(t)^{*} = f_{an}N(t) = f_{an}\,\frac{S(t)}{S_R}\,. \qquad (2)$$

This relationship allows us to describe the excitation function of the fixed selective network as follows

$$e(t) = E \cos \theta = E \cos \left(\frac{d\theta}{dt}\right)t = E \cos \,[2\pi f_{bn}(t)]t \qquad (3)$$

where E = constant magnitude

θ = angular displacement

The Minimum-Analysis-Time Solution

The initial step in the logical development of $N(t)$ is the determination of a minimum-analysis-time solution. This relationship is to be fixed only by the characteristics of the selective network. Each derived component, f_{bn}, is to remain within the pass-band for the minimum time for acceptable analysis. This condition is based on the requirement that the output of the selective system remain within

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*Where subscript n takes on values 0, 1, 2, ...

certain tolerable limits of amplitude and frequency distortion.

Absolute minimum analysis time would result if: (1) $\frac{df_{bn}}{dt}$ equals a constant within the pass-band for all values of $f_{bn}(t)$, and (2) this constant sweep rate is critical for the selective network consistent with tolerable amplitude and frequency distortion.[19]

In the treatment which follows, $N(t)$ is evaluated by a qualitative approach initially. Finally, the nature and magnitude of the errors involved will be determined.

For an ideal, band-pass selective network, the build-up time, τ, is related to the width of the resonant response curve at the cut-off requencies, Δf, as follows[8]

$$\tau = \frac{1}{\Delta f} \quad . \tag{4}$$

Let us assume that the time, Δt, any component f_{bn} must remain within the pass-band is proportional to the build-up time.

$$\Delta t = K\tau = \frac{K}{\Delta f} \tag{5}$$

where

K = a pure numeric (cycles)

This certainly will provide a specified minimum distortion, providing K is made large enough to satisfy the critical excitation sweep rate. Whether this relationship will provide the absolute-minimum-analysis-time solution depends upon the sweep linearity within the pass-band.

Divide both sides of Equation 5 by Δf and invert.

Then,

$$\frac{\Delta f}{\Delta t} = \frac{(\Delta f)^2}{K} \quad . \tag{6}$$

The fundamental multiplier relationship has been expressed as

$$f_{bn}(t) = f_{an}N(t) = f_{an}\frac{S(t)}{S_R} \; . \tag{2}$$

Differentiate Equation 2, considering f_{an} constant.

$$\frac{df_{bn}}{dt} = \frac{f_{an}}{S_R}\frac{dS}{dT} \tag{7}$$

For this analyzer

$$fS = f_u S_R = f_o S_o = \text{constant} \tag{8}$$

where

f = the recorded component, f_{an}, being analyzed at time t (when $f_{bn}(t) = f_u$).

S = the value of $S(t)$ at time t.

f_l = the lower cut-off frequency of the selective network.

f_u = the upper cut-off frequency of the selective network.

f_o = the value of f at $t = 0$ (hence, $f_o = f_{ao}$).

S_o = the value of S at $t = 0$.

Therefore, Equation 7 can be expressed as follows

$$\frac{df_{bn}}{dt} = \frac{f_u}{S}\frac{dS}{dt} \;. \tag{9}$$

Let us assume that

$$\frac{\Delta f}{\Delta t} \cong - \frac{df_{bn}}{dt} \tag{10}$$

We can then equate Equations 6 and 9, and integrate

$$\int_o^t dt = \frac{K}{(\Delta f)^2} f_u \int_{S(t)}^{S_o} \frac{dS}{S}$$

$$t = \frac{Kf_u}{(\Delta f)^2} \ln \frac{S_o}{S} = \frac{1}{c} \ln \frac{S_o}{S} \tag{11a}$$

or,

$$S(t) = S_o\ e^{-ct} \qquad (11b)$$

where $\qquad c = \frac{(\Delta f)^2}{Kf_u}$

Furthermore, use of Equation 8 allows this relationship to be expressed as

$$t = \frac{Kf_u}{(\Delta f)^2}\ \ln\ \frac{f}{f_o} = \frac{1}{c}\ \ln \frac{f}{f_o} \qquad (12)$$

Equations 11b and 12 have been non-dimensionalized and plotted in Figure 3.1.

We can now determine the error involved in the approximation of Equation 10. The general expression for $\frac{df_{bn}}{dt}$ is obtained from Equation 7. Specifically,

$$\frac{df_{bn}}{dt} = - \frac{cf_{sn}S_o e^{-ct}}{S_R} = - cf_{bn}(t) \qquad (13)$$

Let us investigate the specific region of interest: $f_l < f_{bn} < f_u$.

$$\left.\frac{df_{bn}}{dt}\right]_{f_{bn}=f_u} = - cf_u = - \frac{(\Delta f)^2}{K}$$

But, this equals $- \frac{\Delta f}{\Delta t}$ from Equation 6.

Hence

$$\left.\frac{df_{bn}}{dt}\right]_{f_{bn}=f_u} = - \frac{\Delta f}{\Delta t}$$

It can be seen that the maximum error involved in the original assumption occurs when $f_{bn}(t) = f_l$. From Equation (13)

$$\left.\frac{df_{bn}}{dt}\right]_{f_{bn}=f_l} = - cf_l\ .$$

The maximum deviation from linearity can be expressed as

$$(\text{Error})_{\text{Max}} = \frac{\left.\frac{df_{bn}}{dt}\right]_{f_{bn}=f_u} - \left.\frac{df_{bn}}{dt}\right]_{f_{bn}=f_l}}{\left.\frac{df_{bn}}{dt}\right]_{f_{bn}=f_u}} = \frac{f_u - f_l}{f_u} \; . \qquad (14)$$

Since this system will employ a high-Q selective network, the approximation of Equation 10 is a valid one.

Two characteristics of the sweep rate for the minimum-analysis-time solution should be noted. First, when $f_{bn}(t)$ equals f_u, the maximum value of $\frac{df_{bn}}{dt}$ occurs. Furthermore, this maximum value is constant, independent of the value of f_{bn}. Therefore, if the sweep rate of the derived components is less than a certain critical value when $f_{bn}(t)$ equals f_u, tolerable amplitude and frequency distortion will exist throughout the analysis. Secondly, $\frac{df_{bn}}{dt}$ is approximately equal to $-\frac{\Delta f}{\Delta t}$ within the pass-band of the selective network. These two quantities are exactly equal at the upper cut-off frequency, and have maximum deviation at the lower cut-off frequency. The maximum deviation from linear frequency sweep is slightly less than $\frac{1}{Q}$,

where

$$Q = \frac{f_f}{f_u - f_l} = \frac{f_f}{\Delta f}$$

f_f = Mid band frequency of selective network

A Linear Multiplier

To assist in the selection of N(t), a linear speed-time relationship will be investigated. Let us continue the intuitive analysis of Chapter II-A. Figures 2.2 and 2.3 represent the general physical relationships involved. These can be replaced by Figures 3.1 and 3.2 which describe a specific linear multiplier. Note that for this case

FIGURE B.1

A LINEAR SPEED-TIME RELATIONSHIP, N(t)

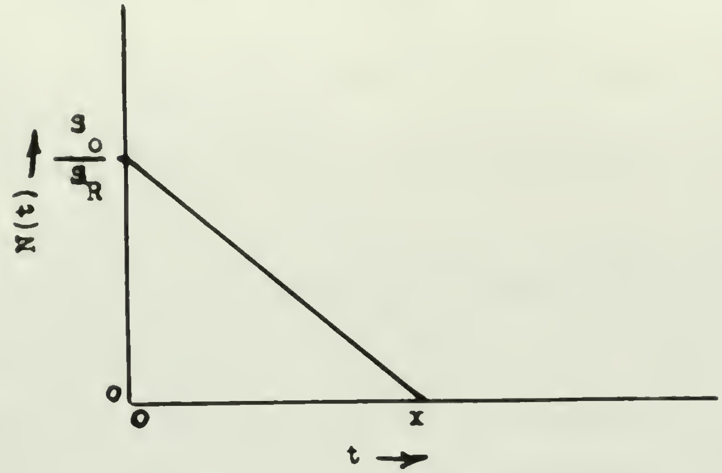
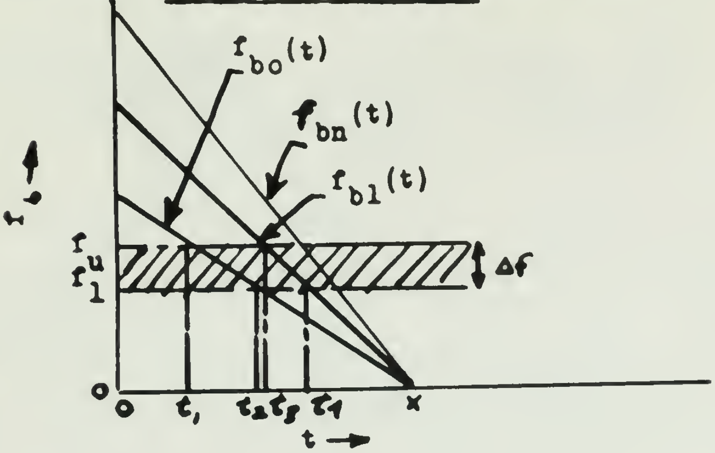


FIGURE B.2

LINEAR FREQUENCY MULTIPLIER OUTPUT AS A FUNCTION OF TIME AND RECORDED FREQUENCY COMPONENT


$f_{bo}(t)$
$f_{bn}(t)$
$f_{bl}(t)$
f_b
f_u
f_l
Δf
0
0
t_1
t_2
t_3
t_4
x
t

$f_{bn}(t)$ is always linear. However, its slope is not of constant magnitude, but is a function of the value of the recorded component being analyzed. That is,

$$\frac{df_{bo}}{dt} < \frac{df_{b1}}{dt} < \frac{df_{bn}}{dt}_{max}$$

Although $\left(\frac{df_{bn}}{dt}\right)_{max}$ is the critical sweep rate, all other values of $\frac{df_{bn}}{dt}$ will oversatisfy the characteristic time requirements of the selective network.

We are interested in determining the function N(t) which will satisfy the following requirements:

(1) It must be a linear speed-time relationship.

(2) $\left(\frac{df_{bn}}{dt}\right)_{max}$ must equal the critical sweep rate of the selective network.

From Equations 7 and 13,

$$\left(\frac{df_{bn}}{dt}\right)_{max} = \frac{(f_{an})_{max}}{S_r} \frac{dS}{dt} = -cf_u$$

Hence,

$$\int_{S_o}^{S(t)} dS = -\frac{cf_u S_r}{(f_{an})_{max}} \int_o^t dt$$

and,

$$S(t) = S_o - \frac{cf_u S_r}{(f_{an})_{max}} t = S_o \left[1 - \frac{cf_o t}{(f_{an})_{max}} \right]. \qquad (15a)$$

where

$$S_o = \frac{f_u S_r}{f_o} .$$

This can be re-expressed as

$$t = \frac{(f_{an})_{max}}{cf_o} \left[1 - \frac{S(t)}{S_o} \right]. \qquad (15b)$$

Also, from Equation (8),

$$t = \frac{(f_{an})_{max}}{cf_o}\left[1 - \frac{f_o}{f} \right]. \tag{15c}$$

A non-dimensionalized plot of Equation 15 is presented in Figure 3.1.

The Equal-Sample-Analysis Solution

The second step in the formulation of an appropriate multiplier function is to consider those desirable features which might be incorporated in this new analyzer. Previously we have only concerned ourselves with the limitations of the selective network in order to obtain maximum resolution within minimum analysis time. In addition it is desirable that each analyzed component remain under scrutiny for the entire length of sample. This requirement would result in the energy reported at each frequency being associated with the same analysis sample as the energies reported for all other frequencies. Currently available types of wave analyzers do not accomplish this feat. The possible ambiguity in measurements which can result is discussed in Appendix C.

Equal sample analysis can be described as

$$S_o \Delta t_o = S \Delta t = \text{constant}. \tag{16}$$

where

Δt_o = the time component f_{bo} must remain within the pass-band.

In the minimum-analysis-time solution, Δt is constant; however, S is continually decreasing. The analysis of the lowest frequency component traverses the entire length of sample. On the other hand, higher frequency analyses cover progressively smaller portions of the original sample. This can lead to an ambiguity in measurements as discussed in Appendix C. A similar appraisal of the linear multiplier relationship

indicates that it also presents unequal sample length analyses.

It is possible to evaluate N(t) in the qualitative fashion which has been followed previously. Equations 5, 8, and 16 can be combined to yield

$$\Delta t = \frac{S_o \Delta t_o}{S} = \frac{f}{f_o} \Delta t_o = \frac{K}{\Delta f} \frac{f}{f_o} \qquad (17)$$

Divide both sides of the equation by Δf and invert.

$$\frac{\Delta f}{\Delta t} = \frac{(\Delta f)^2 f_o}{Kf} = \frac{(\Delta f)^2 S}{KS_o} \qquad (18)$$

Let us assume that

$$\frac{\Delta f}{\Delta t} \simeq - \frac{df_{bn}}{dt} . \qquad (19)$$

From Equation (9) and (18),

$$\frac{df_{bn}}{dt} = \frac{f_u}{S} \frac{dS}{dt} = - \frac{(\Delta f)^2 S}{KS_o} \qquad (20)$$

Integrating,

$$- \int_{S_o}^{S(t)} \frac{1}{S^2} dS = \frac{(\Delta f)^2}{KS_o f_u} \int_o^t dt$$

or

$$t = \frac{Kf_u}{(\Delta f)^2} \left(\frac{S_o}{S} - 1 \right) = \frac{1}{c} \left(\frac{S_o}{S} - 1 \right) \qquad (21a)$$

Therefore,

$$S(t) = \frac{S_o}{1+ct} . \qquad (21b)$$

Similarly,

$$t = \frac{1}{c} \left(\frac{f}{f_o} - 1 \right) . \qquad (21c)$$

A non-dimensionalized plot of Equation 21 is presented in Figure 3.1.

It is now possible to examine the error resulting from the approximation of Equation 19. Differentiate Equation 21b.

$$\frac{dS(t)}{dt} = -\frac{cS_o}{(1+ct)^2} = -\frac{cS(t)}{1+ct}$$

This expression can be substituted in Equation 7 as follows

$$\frac{df_{bn}}{dt} = -\frac{f_{an}}{S_r}\frac{(cS)}{(1+ct)} = -\frac{cf_{bn}(t)}{1+ct} \qquad (22)$$

Let us examine the region of interest: $f_1 < f_{bn} < f_u$.

$$\left.\frac{df_{bn}}{dt}\right]_{f_{bn}=f_u} = -\frac{cf_u}{1+ct} = -\frac{cf_u S}{S_o} = -\frac{(\Delta f)^2 S}{KS_o}$$

But, this equals $-\frac{\Delta f}{\Delta t}$ from Equation 18.

Hence,

$$\left.\frac{df_{bn}}{dt}\right]_{f_{bn}=f_u} = -\frac{\Delta f}{\Delta t}$$

As in the minimum-analysis-time solution, the maximum deviation from linearity occurs at the lower cut-off frequency of the selective network. From Equation 22

$$\left.\frac{df_{bn}}{dt}\right]_{f_{bn}=f_1} = -\frac{cf_1}{1+ct} \ .$$

The maximum deviation from linearity is expressed as

$$(\text{Error})_{max} = \frac{\left.\frac{df_{bn}}{dt}\right]_{f_{bn}=f_u} - \left.\frac{df_{bn}}{dt}\right]_{f_{bn}=f_1}}{\left.\frac{df_{bn}}{dt}\right]_{f_{bn}=f_u}} = \frac{f_u - f_1}{f_u} \qquad (23)$$

Hence, within the pass-band the maximum deviation from linearity is the same for both the minimum-analysis-time solution and the equal-sample-analysis solution. Its maximum magnitude is less than $\frac{1}{Q}$. For all practical purposes, the sweep rate is linear since the analyzer

system will consist of a high-Q selective network.

Figure 2.3 presents a general picture of the family of decaying components, $f_{bn}(t)$, sweeping across a fixed pass-band. The minimum-analysis-time solution exhibits critical sweep rates at f_u for all values of $f_{bn}(t)$. On the other hand, the equal-sample-analysis solution has only one critical sweep rate: this occurs for the derived component $f_{bo}(t)$ associated with the lowest recorded frequency component, f_{ao}. All other values of $f_{bn}(t)$ will oversatisfy the characteristic time requirements of the selective network.

The Excitation Function, e(t)

The excitation function of the selective network, e(t), has been described previously as

$$e(t) = E \cos \theta = E \cos (d\theta/dt)t = E \cos [2\pi f_{bn}(t)]t = E \cos [\omega_{bn}(t)]t \quad (3)$$

From equation (2)

$$f_{bn}(t) = f_{an}N(t) = f_{an} \frac{S(t)}{S_r}$$

Table B.1 relates the essential characteristics of e(t) resulting from the three different multiplier relationships. In order to complete the usefulness of this table, the values of the angular frequency sweep rate $\frac{d\omega_{bn}}{dt}$ and the ratio $\frac{df_{bn}/dt}{f_{bn}}$ are provided. Since we are only interested in the critical rate for the linear multiplier, its tabulated characteristics pertain only to $(f_{an})_{max} = f_c$. Note that, wherever possible, reference is made in the table to equations derived in the text.

It will be shown that it is useful to express θ by a Maclaurin's Series for the short time interval about $t = 0$. Three such series are presented in Table B.2.

TABLE B.1

CHARACTERISTICS OF THREE MULTIPLIER SPEED-TIME RELATIONSHIPS

	Minimum-Analysis-Time Solution	Linear Multiplier*	Equal-Sample-Analysis Solution
$S(t)$	$S_o e^{-ct}$ [Eq. 11b]	$S_o(1-ct)$ [Eq. 15a]	$\frac{S_o}{1 + ct}$ [Eq. 21b]
$\omega_{bn}(t) = \frac{d\theta}{dt}$	$\omega_{bno} e^{-ct}$	$\omega_{bno}(1-ct)$	$\frac{\omega_{bno}}{1 + ct}$
θ	$\frac{\omega_{bno}}{c}(1-e^{-ct}) + \epsilon$	$\frac{-c\omega_{bno}t^2}{2} + \omega_{bno}t + \epsilon$	$\frac{\omega_{bno}}{c} \ln(1+ct) + \epsilon$
$\frac{d\omega_{bn}}{dt}$	$- c\omega_{bno} e^{-ct}$ [Eq. 13]	$- c\omega_{bno}$	$\frac{- c\omega_{bno}}{(1+ct)^2}$ [Eq. 22]
$\frac{d\omega_{bn}}{\omega_{bn}}$	$- c$	$\frac{-c}{1-ct}$	$\frac{-c}{1+ct}$

where $\omega_{bno} = \frac{2\pi f_{an} S_o}{S_r}$; $c = \frac{(\Delta f)^2}{Kf_u}$; ϵ = the value of θ at t equals 0

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*with $(fan)_{max} = f_o$

TABLE B.2

θ EXPRESSED AS A MACLAURIN'S SERIES

Linear Multiplier: $\theta = \epsilon + \omega_{bno} t - \frac{c\omega_{bno} t^2}{2}$

Minimum-Analysis-Time Solution: $\theta = \epsilon + \omega_{bno} t - \frac{c\omega_{bno} t^2}{2} + \frac{c^2\omega_{bno} t^3}{6} - \cdots$

Equal-Sample-Analysis Solution: $\theta = \epsilon + \omega_{bno} t - \frac{c\omega_{bno} t^2}{2} + \frac{c^2\omega_{bno} t^3}{3} - \cdots$

TABLE B.3

TRIGONOMETRIC IDENTITIES

1. $\sin(\alpha - 90) = -\cos\alpha$
2. $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
3. $\cos\alpha\cos\beta = \frac{1}{2}\cos(\alpha - \beta) + \frac{1}{2}\cos(\alpha + \beta)$
4. $\sin\alpha\cos\beta = \frac{1}{2}\sin(\alpha + \beta) + \frac{1}{2}\sin(\alpha - \beta)$
5. $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

Theoretical Evaluation of the System Response

Reference has been made to a requirement that the output of the selective system remain within certain tolerable amplitude and frequency distortion. From Table B.1 the critical sweep rate for three possible relationships for N(t) can be expressed as

$$\left(\frac{df_{bn}}{dt}\right)_{critical} = - cf_u = - \frac{(\Delta f)^2}{K} \; .$$

or

$$K = - \frac{(\Delta f)^2}{\left(\frac{df_{bn}}{dt}\right)_{critical}} = \frac{(\Delta f)^2}{\left(\frac{df_{bn}}{dt}\right)_{critical}} \qquad (24)$$

It will be shown that the amount of distortion present is a function of the value of K. In order to evaluate this distortion, it is necessary to determine the system response to an excitation function. This can be accomplished by application of the superposition theorem[7,8] which describes the response of a linear system to an arbitrary excitation function in terms of the response of the system to a unit impulse. This theorem takes the form of the real convolution integral

$$r(t) = \int_{-\infty}^{t} e(\tau)\, h(t-\tau)\, d\tau \qquad (25)$$

where

r(t) = the response of the system.

e(t) = the excitation function of the system.

h(t) = the unit impulse response of the linear system.

The analytic treatment of this integral can turn out to be extremely difficult or practically impossible. However, a graphical treatment can be applied in such cases. Gardner and Barnes[7] describe the basic

procedures involved. Such an evaluation is very useful, but it is also very time consuming.

The following analysis is based upon a system which obeys a linear, second order differential equation. To assist in the analytic treatment of Equation 25, the specific LCR circuit of Figure B.3 has been selected. Similar results would be obtained by the use of any linear second order system. The impulse response can be determined from Equation 26 in Figure B.6 by use of Laplace Transform 1.303 from Gardner and Barnes.[7]

$$h(t) = \frac{\omega_o e^{-\omega_o t/2Q} \sin [\omega_o \sqrt{1 - \frac{1}{4Q^2}}\, t - \psi]}{Q\sqrt{1 - \frac{1}{4Q^2}}} \tag{27}$$

or, alternatively

$$h(t) = \frac{\omega_o e^{-\omega_o k_1 t}}{Qk_2} \quad \sin (\omega_o k_2 t - \psi) \tag{27a}$$

where

$$\psi = \tan^{-1} \sqrt{4Q^2-1}$$

$$k_1 = \frac{1}{2Q}$$

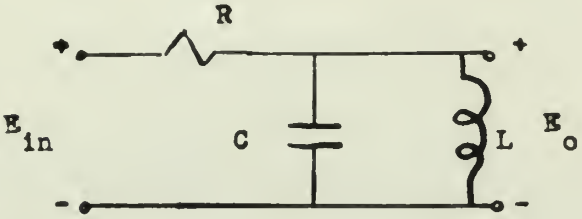
$$k_2 = \sqrt{1 - k_1{}^2}$$

In order to evaluate convolution equation 25, it is necessary to describe the excitation function. Table B.2 indicates that for the two non-linear multiplier functions, the first three terms in a Maclaurin's Series of θ are identical to a linear frequency sweep. Furthermore, it has been shown that these functions exhibit very little deviation from a linear frequency sweep. (A high-Q selective network exists.) Therefore, the excitation functions can be represented as

$$e(t) \simeq E \cos (\epsilon + \omega_{bmo} t - \omega_{bmo} \frac{ct^2}{2}) \tag{28}$$

Figure B.3

The Selective System Used For Response Evaluation



$$E_o(s) = \frac{E_{in}(s)}{1 + RCs + \frac{R}{Ls}}$$

$$= \frac{s\,E_{in}(s)}{RC\left[\,s^2 + \frac{s}{RC} + \frac{1}{LC}\,\right]}$$

therefore,

$$E_o(s) = \frac{\omega_o \quad s\,E_{in}(s)}{Q\ [s + \omega_o/\,2Q]^2 + [\,\omega_o\sqrt{1 - 1/4Q^2}\,]^2} \ldots (26)$$

where

$$Q = \frac{R}{L\omega_o}$$

$$\omega_o = \frac{1}{\sqrt{LC}}$$

Ein(s) = 1 for unit impulse

Substitute e(t) and h(t) into Equation 25.

$$r(t) = \frac{\omega_o E}{k_2 Q} \int_{-\infty}^{t} \cos \left(\epsilon + \omega_{bno} \tau - \frac{c\omega_{bno}}{2} \tau^2 \right) e^{-\omega_o k_1 (t-\tau)} \sin \left[\omega_o k_2 (t-\tau) - \psi \right] d\tau \quad (29)$$

For derivation purposes it is not necessary to terminate the series expansion in θ to a t^2 term as is done in Equation 28. Hok[10] presents a general procedure for solving any transient frequency-modulated signal that can be represented by an excitation vector of the form

$$e = E \exp \left[j \left\{ \omega_o t + f(t) \right\} \right]$$

However, this method requires extensive use of the Fresnel integral, which has not been tabulated to any appreciable extent for complex variables. On the other hand, once Equation 28 is limited to the indicated series expansion, we are confining ourselves to the study of a linear-frequency-sweep excitation. The basic evaluation problem posed by this type of excitation for a simple selective network is not a new one. We attempted several, seemingly-original approaches to the breakdown of Equation 29. However, closer examination of our solutions indicated otherwise. Each approach was merely a variation of one or more procedures contained in recent literature.* One of these solutions is presented below in order to provide a better understanding of the basic phenomena involved. This approach represents a modification of Barber and Ursell's[2] competent study of a mechanical analogy to our system, which is a variable-speed optical analyzer employing a resonant vibration galvanometer. These authors provide two methods for evaluating Equation 29. The first is exhaustive and determines an upper limit to the errors involved. Although

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*These papers are reviewed in Chapter I.

the second method is more superficial, it suggests a physical picture of the response. For this reason the following analysis parallels the latter approach to the problem.

For our wave analyzer Q is very large. For this case

$$\psi \simeq 90°$$

$$k_2 \simeq 1$$

Therefore, from Trigonometric Identity 1*

$$h(t) \simeq \frac{-\omega_o e^{-\omega_o k_1 t}}{Qk_2} \quad \cos \omega_o k_2 t$$

Let

$$\omega_{bmo} = \omega_o = \omega$$

$$A = -E$$

Next, the excitation e(t), whose frequency is slowly changing, may be regarded as the sum of two components of constant frequency, $\frac{\omega}{2\pi}$, whose amplitudes are slowly changing. From Trigonometric Identity 2, equation 28 can be represented as

$$e(t) = A \cos (\epsilon + \omega t) \cos (\frac{c\omega t^2}{2}) + A \sin (\epsilon + \omega t) \sin (\frac{c\omega t^2}{2})$$

The effects of these two components will be considered by separate substitution into Convolution Equation 25.

Then

$$r_1(t) = \frac{\omega A}{Q}\int_{-\infty}^{t} e^{-\omega k_1(t-\tau)} \cos (\frac{c\omega\tau^2}{2}) \cos (\epsilon + \omega\tau) \cos \omega(t - \tau) d\tau$$

Identity 3 allows this to be expressed as

$$r_1(t) = \frac{\omega A}{2Q}\int_{-\infty}^{t} e^{-\omega k_1(t-\tau)} \cos (\frac{c\omega\tau^2}{2})[\cos (\epsilon + \omega t) + \cos (\epsilon + 2\omega\tau - \omega t] d\tau \quad (30)$$

Only the second term within the brackets of Equation 30 varies rapidly with . If we write for this term its mean value of zero,

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*All trigonometric identities are listed in Table B.3.

$$r_1(t) = \frac{\omega A}{2Q} \cos(\epsilon + \omega t)\, e^{-\omega k_1 t} \int_{-\infty}^{t} e^{\omega k_1 \tau} \cos\left(\frac{c\omega\tau^2}{2}\right) d\tau \qquad (31)$$

or

$$r_1(t) = \frac{\omega A}{Q} \cos(\epsilon + \omega t)\, X_1 \qquad (31a)$$

Similarly,

$$r_2(t) = \frac{\omega A}{Q} \int_{-\infty}^{t} e^{-\omega k_1 (t-\tau)} \sin\left(\frac{c\omega\tau^2}{2}\right) \sin(\epsilon + \omega\tau) \cos\omega(t-\tau)\, d\tau$$

By Identity 4

$$r_2(t) = \frac{\omega A}{2Q} \int_{-\infty}^{t} e^{-\omega k_1 (t-\tau)} \sin\left(\frac{c\omega\tau^2}{2}\right) [\sin(\epsilon + \omega t) + \sin(\epsilon + 2\omega\tau - \omega t)]\, d\tau \qquad (32)$$

Using the same argument as in Equation 31 above,

$$r_2(t) = \frac{\omega A}{2Q} \sin(\epsilon + \omega t)\, e^{-\omega k_1 t} \int_{-\infty}^{t} e^{\omega k_1 \tau} \sin\left(\frac{c\omega\tau^2}{2}\right) d\tau \qquad (33)$$

or

$$r_2(t) = \frac{\omega A}{Q} \sin(\epsilon + \omega t)\, Y_1 \qquad (33a)$$

where X_1 and Y_1 are factors which can be considered as amplitudes which vary slowly with time.

The above expressions can be combined to yield

$$r(t) = r_1(t) + r_2(t)$$

$$= \frac{\omega A}{Q} [X_1 \cos(\epsilon + \omega t) + Y_1 \sin(\epsilon + \omega t)] \qquad (34)$$

where

$$\left.\begin{matrix} X_1 \\ \\ Y_1 \end{matrix}\right\} = \frac{1}{2} \int_{-\infty}^{t} e^{-\omega k_1 (t-\tau)} \left.\begin{matrix} \cos \\ \\ \sin \end{matrix}\right\} \left(\frac{c\omega\tau^2}{2}\right) d\tau \qquad (34a)$$

If we change variables so that

$$u = t - \tau$$

then

$$\left.\begin{matrix} X_1 \\ \\ Y_1 \end{matrix}\right\} = \frac{1}{2}\int_0^{\infty} e^{-\frac{\omega u}{2Q}} \left.\begin{matrix} \cos \\ \\ \sin \end{matrix}\right\} \frac{1}{2}\, c\omega\, (t - u)^2 du \qquad (34b)$$

Equation 34 relates the resonant frequency and the instantaneous frequency of oscillation. It is also convenient to study the excitation frequency, $\frac{1}{2\pi}\frac{d\theta}{dt}$. We can define two new variables X and Y where

$$X_1 = -[X \cos \tfrac{1}{2} c\omega t^2 + Y \sin \tfrac{1}{2} c\omega t^2]$$

$$Y_1 = - X \sin \tfrac{1}{2} c\omega t^2 + Y \cos \tfrac{1}{2} c\omega t^2 \qquad (35)$$

If these relationships are substituted into Equation 34 we see from Identities 2 and 5 that

$$r(t) = \frac{\omega A}{Q}[Y \sin (\epsilon + \omega t - \frac{c\omega t^2}{2}) - X \cos (\epsilon + \omega t - \frac{c\omega t^2}{2})]$$

$$= \frac{\omega A}{Q}[Y \sin \theta(t) - X \cos \theta(t)] \qquad (36)$$

where

$$X = \frac{1}{2}\int_0^{\infty} e^{-\frac{\omega u}{2Q}} \cos (c\omega tu - \tfrac{1}{2} c\omega u^2)\, du$$

$$Y = \frac{1}{2}\int_0^{\infty} e^{-\frac{\omega u}{2Q}} \sin (c\omega tu - \tfrac{1}{2} c\omega u^2) du \qquad (36a)$$

Examination of Equation 36 indicates that the system response is the sum of two oscillations whose frequencies vary like the excitation frequency and whose amplitudes vary slowly with time. In equation 34 we see that

$$\frac{1}{2\pi}\ \frac{d}{dt}\left[\tan^{-1} \frac{X_1}{Y_1}\right]$$

is a measure of the difference between the resonant frequency and the instantaneous frequency of oscillation. On the other hand, Equation 36 illustrates that the difference between the excitation frequency and the instantaneous frequency of oscillation can be indicated by

$$\frac{1}{2\pi}\frac{d}{dt}\left[\tan^{-1}\frac{X}{Y}\right] .$$

Expressions 34 and 36 are equivalent. Both lead to the same value for the amplitude of the response, R.

$$R = [X^2 + Y^2]^{1/2} = [X_1{}^2 + Y_1{}^2]^{1/2}$$

With the aid of the Admiralty Computing Service, Barber and Ursell have plotted envelopes of this transient resonance.[2,3] Their results, with appropriate changes in notation, are presented in Chapter II-A.

Appendix C

AMBIGUITY IN WAVE ANALYZER MEASUREMENTS*

It has, for some time, been recognized that spectrum analyses made by currently available types of wave analyzer apply to wave energy which is varying in character during the analysis. In other words, the energy reported at one frequency is not associated with the same sample of the phenomena analyzed as the energies reported for other frequencies. It is the purpose of this memorandum to examine briefly the nature and magnitude of the errors likely to appear in sound analyses made by instruments of this type.

The oscillograph forming part of Figure C.1 shows, as a function of time, the output of a band-pass filter when responding to the underwater sounds due to the propeller of a passing ship. The output of this filter is restricted to components the frequencies of which lie in the half octave between 212 and 300 cycle/sec. An examination of the outputs of filters passing bands on either side of this indicates that during those time intervals for which a large response is reported the energy spectrum for this band may be virtually continuous and that the energy distribution may be nearly uniform. There is, of course, no assurance of this; a trace having this general appearance would result from a component having a nominal frequency of 250 cycle/sec, but modulated as indicated by the envelope of the trace.

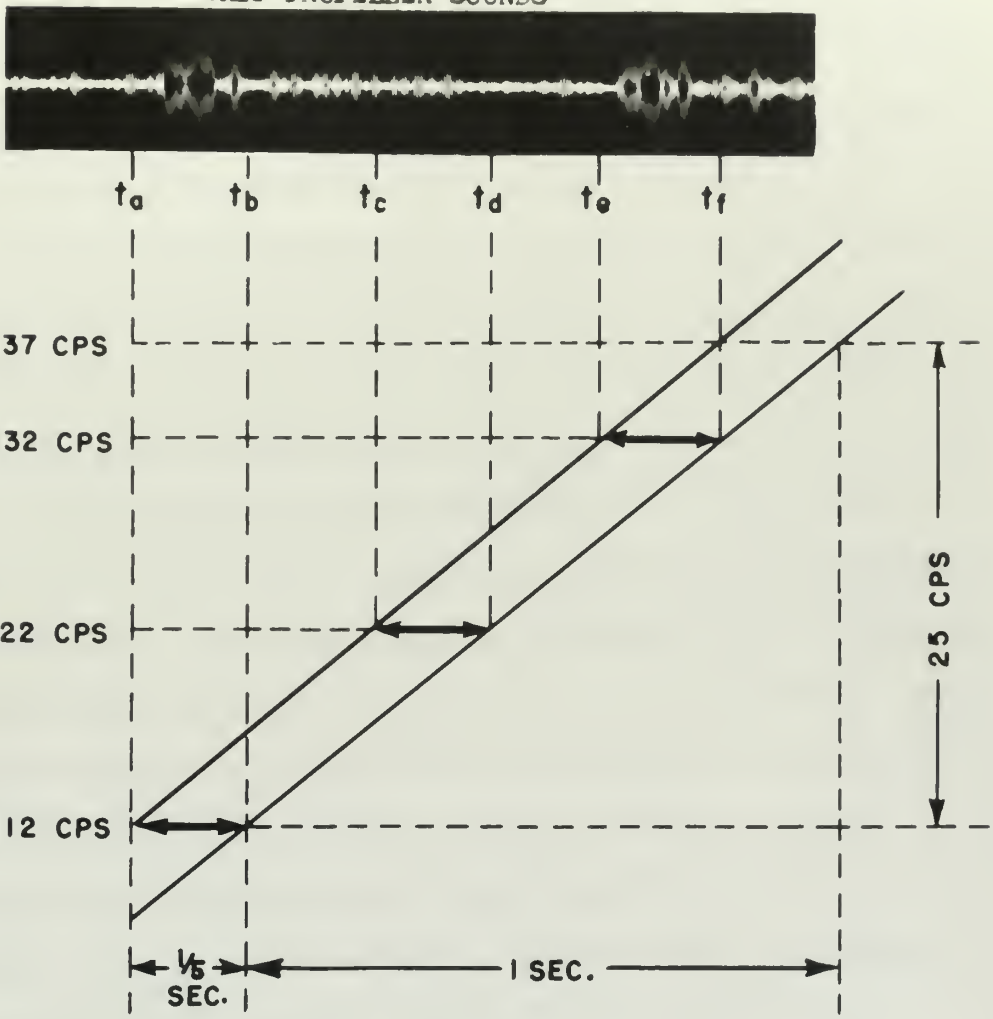
For the frequency range here under consideration it is customary to use an analyzer having a fixed band width of $\Delta f = 5$ cycles/sec. If this system is to respond properly to any change in energy level it is

---

*The discussion which follows is due to Doctor J. W. Horton of the U.S. Navy Underwater Sound Laboratory, Fort Trumbull, New London, Connecticut. (Reference 11.)

FIGURE C.1

RESPONSE OF BAND-PASS FILTER TO UNDERWATER SHIP PROPELLER SOUNDS


t_a
t_b
t_c
t_d
t_e
t_f
237 CPS
232 CPS
222 CPS
212 CPS
25 CPS
1/5 SEC.
1 SEC.

U. S. Navy Underwater Sound Laboratory
NP24 - 9641 - 11 - 51

Official Photograph

necessary that each component remain within the pass band for at least $\Delta t = 1/5$ sec. This fixes the rate at which the spectrum may be scanned at $\Delta f/\Delta t = 25$ cycle/sec^2. Now let us assume that the time interval during which the system is responsive to a component having a frequency of 212 cycle/sec is the 1/5 sec between the times t_a and t_b marked on the oscillogram. The time interval during which it would be responsive to a component having a frequency of 222 cycle/sec would then be the 1/5 sec between t_c and t_d. If it is indeed a fact that the increase in total energy shown by the oscillograph to occur between t_a and t_b results from an increase in the energies of all components within the half octave being analyzed the response of the selective system will show an increase during this interval. This increase will be reported as being associated with a frequency of 212 cycle/sec but not with any particular time. If, during this same interval, the system had been responsive to any other frequency within the half octave band it would have shown a similar increase in response. During the time interval between t_c and t_d the selective system will, under the conditions represented by the diagram, show a reduced response. This will be reported as associated with a frequency of 222 cycle/sec. This system would, however, have shown a low response for this time interval if its selective system had been responsive to components at any other portion of the frequency spectrum. In a similar manner the time interval during which the system is responsive to a component having a frequency of 232 cycle/sec is the 1/5 sec between t_e and t_f. If, again, the increased filter output occurring here represents an increase in the amount of energy associated with all components the analyzer will, once more, show an increased response.

Now, however, this will be associated with a frequency of 232 cycle/sec. As the analysis continues, there will be a similar increase in the response; these will be associated with frequencies of 252, 272, and 292 cycles/sec.

The complete analysis, in other words, appears to show that the energy present in the output of the filter is concentrated in five components, spaced by 20 cycle/sec intervals. This may be very far from the truth. In any statement purporting to specify the manner in which energy varies with frequency it is expected that the data associated with one spectrum interval apply to the same sample of the phenomena as do the data associated with other spectrum intervals. In the case here under consideration this would be equivalent to a statement specifying the results of simultaneous observations of the outputs of a series of selective systems, the 5 cycle/sec bands of which cover the entire half octave passed by the original filter. If such observations, or their equivalent, were to be made it might well be found that more energy was associated with a frequency for which the analyzer reported no energy than for a frequency which, by pure chance, happened to be scanned at a time when all components were passing through energy maxima.

In the original oscillogram we have information relative to the time variation in the rate of flow of the total energy for all components within the half octave band. With respect to the energy within successive narrow intervals of the spectrum, however, it is impossible to determine from the data presented by the analyzer to what extent observed changes in energy magnitude are due to changes in frequency or to what extent they are due to changes in time.

In the case postulated above the rate at which pulses of acoustical energy occur may be computed directly from the information obtained

by the sound analyzer. Knowing that the frequency spectrum is scanned at a rate of 25 cycles/sec^2 and observing that response maxima occur every 20 cycle/sec, it is evident that there is a maximum every 0.8 sec. There are, in other words, 75 pulses/min. This is in agreement with a count made against the time scale of the oscillogram.

It is interesting to examine the manner in which the situation described above is affected by a change in the resolving power of the analyzer. Suppose, for example, that the band width of the selective system is increased to $\Delta f = 10$ cycle/sec; the time required for this system to respond to any change in energy level is then $\Delta t = 1/10$ sec and the spectrum may be scanned at a rate of $\Delta f/\Delta t = 100$ cycle/sec^2. Let us assume, as before, that the time interval between two successive energy maxima in the filter output is 0.8 sec. At the increased scanning rate the spectrum interval scanned during this time interval will be 80 cycle/sec. If, therefore, the system is responsive to a frequency of 212 cycle/sec during the time interval coinciding with one energy maximum, it will be responsive to 292 cycle/sec during the time interval coinciding with the next succeeding maximum. The analyzer will now report that the energy output of the filter is concentrated in two components, separated by an 80 cycle/sec spectrum interval. From this it is evident that when the spacing of response maxima is due to a time variation of energy rather than to its frequency distribution, and when the scanning rate is maintained at the maximum value at which the selective system can properly respond to changes in energy level, the length of the spectrum interval reported between response maxima will vary as the square of the band width.

APPENDIX D

SAMPLE LENGTH CONSIDERATIONS

The theoretical length of sample required for proper analysis can be evaluated from relationships formulated in Appendix B.

The theoretical minimum length of sample, L_{min}, is determined by use of Equation 16.

$$L_{min}\ (\text{seconds}) = \frac{S_o \Delta t_o}{S_r} \qquad (37)$$

where

S_o = the multiplier speed at time t equals 0.

Δt_o = the time interval during which component f_{bo} must remain within the pass-band for proper analysis.

S_r = the constant recording speed.

From Equations 4 and 8 we know that

$$\Delta t_o = \frac{K}{\Delta f} \qquad (4)$$

and

$$S_o = \frac{f_u S_r}{f_{ao}} \qquad (8)$$

Substitute these quantities into Equation 37. Then

$$L_{min}\ (\text{seconds} = \frac{K}{f_{ao}} \left(\frac{f_u}{\Delta f}\right) \qquad (38)$$

where

$$K = \frac{(\Delta f)^2}{df_{bn}/dt}$$

f_u = the upper cut-off frequency of the selective network.

f_{ao} = the recorded frequency component being analyzed at time t equals zero.

Δf = the band-width of the selective system.

$\frac{f_u}{\Delta f}$ = a measure of the percentage resolution.*

---

* For a high-Q system $\frac{f_u}{\Delta f} \simeq \frac{f_f}{\Delta f}$. A 1/2 % error is involved in this approxima-

Note that the theoretical minimum length of sample can also be expressed as

$$L_{min}(\text{cycles}) = \frac{K f_u}{\Delta f} \qquad (38a)$$

Assume that a one percent resolution is desired. For a 1 - 1000 cycles per second analysis-band, 100 x K seconds of sample tape is required. However, a 100 - 100,000 cycles per second analysis-band requires only 1 x K seconds length of tape.

An Alternative Approach

The direct approach presented below is based on a procedure suggested by Doctor J. W. Horton. The analysis is included since it tends to tie-together the quantities Δt and L_{min}.

The logarithmic decrement of a resonant system is defined as

$$\delta = \ln \frac{I_1}{I_2}$$

where $\frac{I_1}{I_2}$ is the ratio of the amplitude of any two successive cycles:

Yet, on page 139 of Reference 30 we see that

$$\delta = \frac{\pi}{Q}$$

where

$$Q = \frac{f_f}{\Delta f}$$

By combining,

$$\ln \frac{I_1}{I_2} = \frac{\pi \Delta f}{f_f}$$

Hence,

$$\ln \left(\frac{I_1}{I_2}\right)^n = \ln \frac{I_o}{I_n} = \frac{\pi n \Delta f}{f_f}$$

Here $\frac{I_o}{I_n}$ is the ratio of the amplitude at the beginning of an interval $\frac{n}{f_f} = \tau$ seconds duration to the amplitude at the end of this interval,

Let

$$\frac{n\Delta f}{f_r} = \tau \Delta f = 1 \text{ cycle}$$

That is, consider an interval proportional to the reciprocal of the bandwidth of the selective system

$$\tau = \frac{1}{\Delta f}^{*}$$

Then

$$\ln \frac{I_o}{I_n} = \pi$$

$$I_n = 0.0433\ I_o$$

Considering the build-up of current in a resonant system, the current would reach 95.7% of its final amplitude during this interval.[**]

The number of cycles required for this change is given by

$$\ln \frac{I_o}{I_n} = \frac{\pi n}{Q} = \pi$$

Hence

$$n = Q$$

This shows that the time requirement of Equation 4 is equivalent to saying that the sample must contain Q cycles. Equation 38a is equivalent providing K equals 1.

Butt-weld Considerations

The butt-weld joint influences the sample length as determined in Equation 38. Beranek[4] points out that the butt-weld is usually of insufficient length to effect the analysis providing playback time for the loop is greater than a second or two. We have seen that a certain

---

[*] This is Equation 4 of Appendix B.

[**] An interesting correspondence exists between this result which indicates -0.4 db power error for K = 1, and Figure 3.3 which predicts -0.5 db power error for K = 1.

minimum number of cycles must be considered in order to achieve a proper analysis. The relative location of the joint and the reproduce head for a given analyzed component can prevent achievement of the required Δt. This butt-weld limitation results from a 180 degree phase shift introduced at the joint. Let us arbitrarily increase the sample length by a factor of two; this eliminates the 180 degree phase shift problem. Therefore, let

$$L \text{ (seconds)} = \frac{2\ K\ f_u}{f_{so}\,\Delta f} \qquad (39)$$

$$L \text{ (cycles)} = \frac{2\ K\ f_u}{\Delta f} \qquad (39a)$$

A partial method of circumventing errors of this nature is for the analyzer operator to separately investigate two different sample loops. These samples should be prepared with a 90 degrees phase shift relative to the location of the butt weld. If for a particular frequency, one loop presents a broad-band indication and the other loop exhibits a peak indication, the operator is made aware that a butt wald ambiguity exists for the broad-band indication.

APPENDIX E

DETAILS OF EXPERIMENTAL EQUIPMENT AND PROCEDURE

Introduction

This appendix gives in more detail a description of the experimental equipment. Some of the more pertinent factors in experimental procedure are described. The last part of the appendix discusses other equipment which will accomplish the same functions as the equipment actually used.

Disc and Reproduce Head Assembly

The heart of the problem and of the unique feature of derivation of analyzer frequency by multiplication is embodied in this assembly.* Figure E.1 is a general view of this assembly, showing also the DC driving motor assembly and tachometer. Figure E.2 shows details of the reproduce head mounting. The disc used to transport the tape was 9.00 inches in diameter and was fabricated from brass. The disc had a minimum run-out or eccentricity of .0017 inches as measured by a dial indicator. The minimum used in this connection refers to the fact that various amounts of run-out were possible by varying the relative angular position of the disc and the shaft which turned the disc. Naturally the minimum run-out was used. Three pulleys on the disc shaft and three corresponding pulleys on the driving shaft permitted speed changes of 10:1, 1:1, and 0.5:1 between the driving shaft and the disc.

No little trouble was encountered in attaching the tape to the disc and in setting the face of the reproduce head at the closest possible distance from the magnetic tape. The trouble in connection with attaching the tape stems from the fact that the glue under the tape caused an

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*The basic disc and reproduce head assembly were provided by the Recording Branch, U.S. Navy Underwater Sound Laboratory.

FIGURE 4.1
Disc Assembly

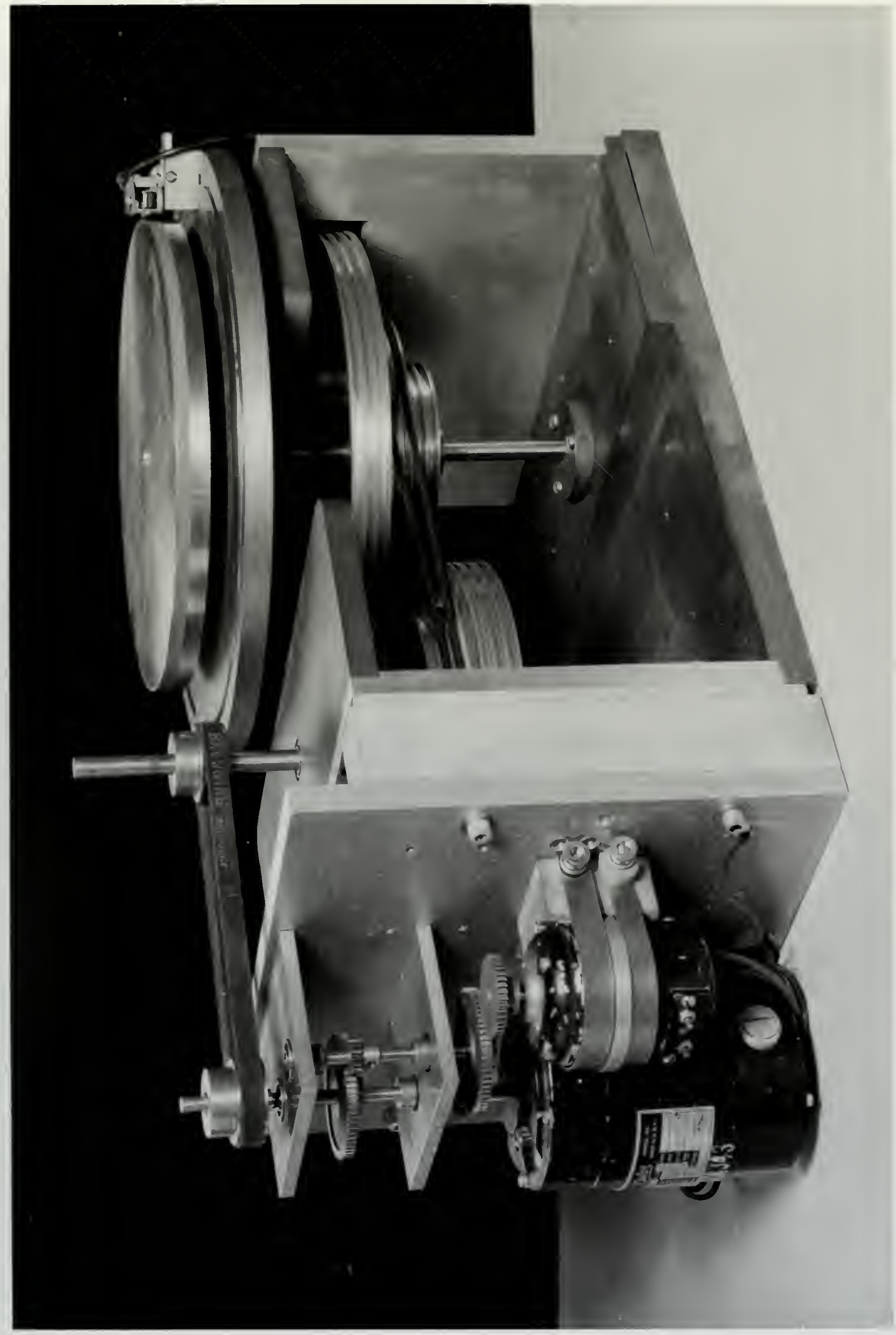
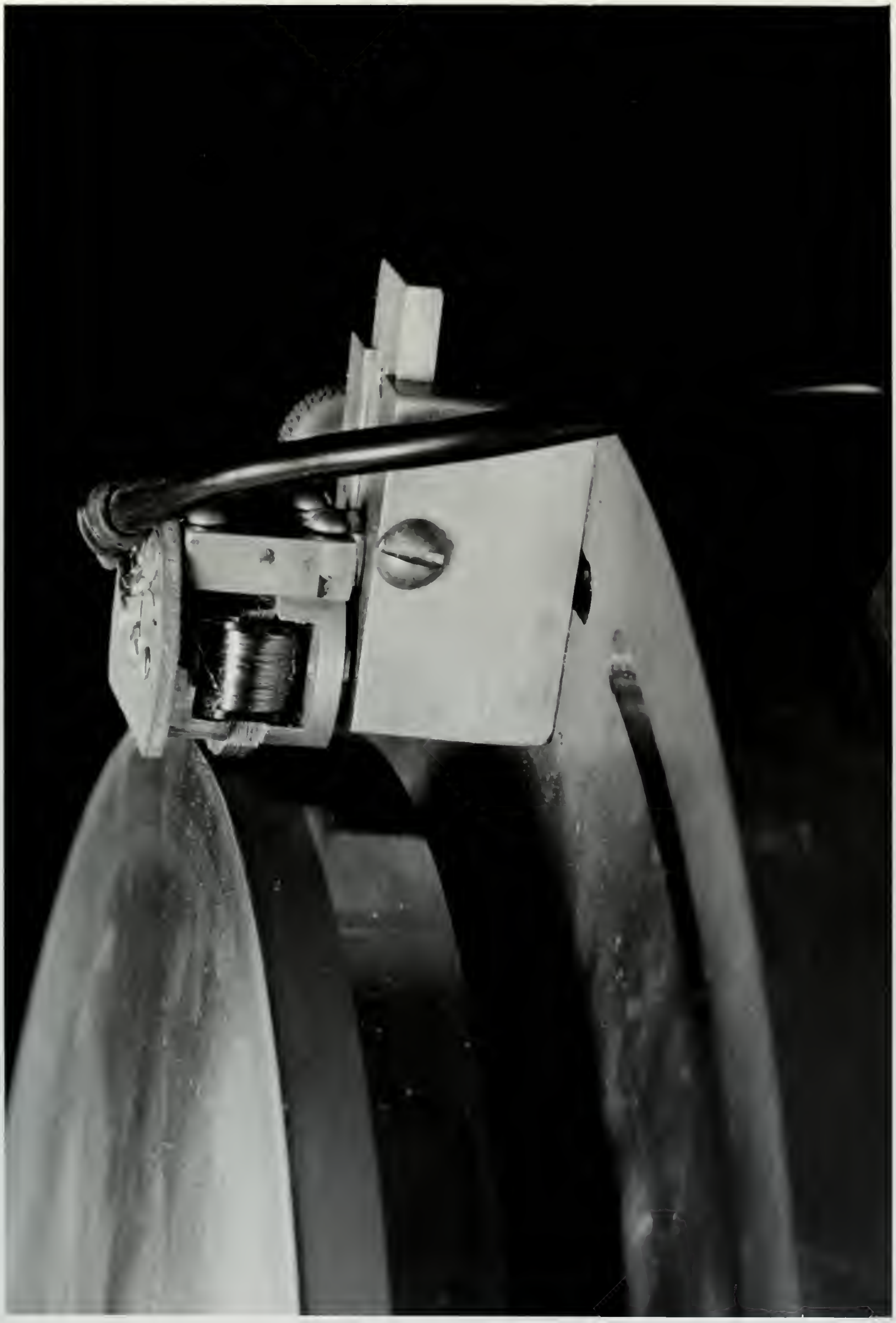


FIGURE 2.9
Temperature Head Assembly



uneven surface of the tape. A number of various glues were used including normal household stationary glue, Duco household cement, leather glue thinned with laquer thinner, and rubber cement. The last glue was by far the best for this use, but even than an uneven surface was obtained. As measured by a dial indicator, irregularities of the order 0.0002 - 0.0004 were obtained. In addition, care had to be taken lest an imperfect bound result in blister like separations of the tape and the disc. These blisters would not show up on the dial indicator but would nevertheless be present, particularly when the disc revolved. The rubber cement was thinned slightly with lacquer thinner in order to obtain a more uniform spreading of the glue, but in such instances the blisters became excessive. A rather uniform thickness of the cement was obtained by revolving the disc while holding a brush wet with glue up against the periphery of the disc. By carefully working the tape from the centers towards the ends, squeezing out the excess glue, and by insuring that there was good adhesion over the entire circumference of the disc, an acceptable bound of the tape to the disc was obtained in the light of the amount of inherent run-out of the disc. The butt joint was placed at the low point of the disc.

Several attempts were made to position the reproduce heads so that it would just touch the tape at the high point of the disc. However, the head tended to tear off the tape when actual contact was made. In addition it was feared that this repetetive solid contact (the tape acting as a poor cushion between the head and disc) might permanently magnetize the reproduce head. Thus a deliberate gap was introduced between the tape on the disc and the reproduce head. This gap was less than 0.0005 inches as indicated by a feeler guage.

The reproduce head used was a Brush Magnetic Recording Company BK - 919A with the following characteristics:

Pole width	.120 inches
Pole face gap	.0005 inches
Total coil resistance	120 ohms
Total impedance at 1800 cycle sec	1950 ohms
Total impedance at 40,000 cycle/sec	37,000 ohms
Maximum output level at 1000 cycle/sec and 7.5 inches/sec	0.004 volts with red ox

The effect of the eccentricity of the disc is shown in an oscilloscope photograph in Figure E.3. This picture is of the output of the tuned circuit for a frequency of 1930 cycles per second. The recorded frequency was 300 cycles per second. It can be seen that the variations in air gap between the reproduce head and the tape cause a variation of about 50% in this case. A more important consideration is that the variation between the high and low points on the disc will in itself vary with the frequency of the recorded component which is being analyzed. The effect of a higher resolution for the system (a higher Q) can be also seen from these photographs. The higher Q filter accentuates the transient nature of this variation. This figure and Figure E.4 show the effect of the butt joint on the response of the filter. The butt joint in this case was about 0.008 inch. In several cases a butt joint was obtained which resulted in an almost imperceptible transient, and when the butt joint was being used to mark the revolutions of the disc, it was necessary to enlarge the width of the butt joint in one instance. The effect of the irregular surface of the tape can be seen from Figure E.5 which is an oscillograph of the recorded signal as amplified. The recorded frequency was 300 cycles/

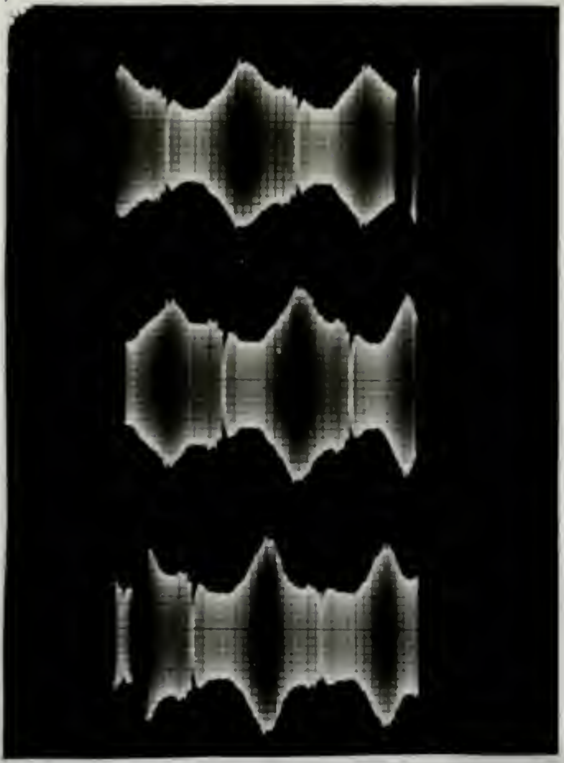


FIGURE E.3
Effect of eccentricity of disc on selected frequency

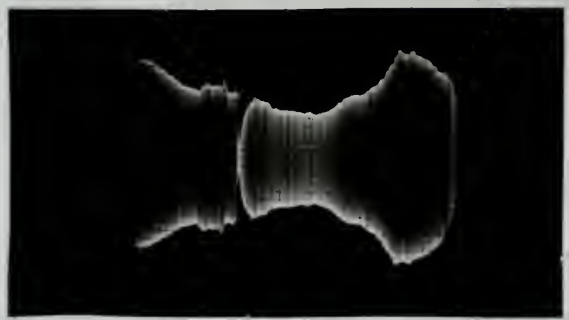


FIGURE E.4
Butt joint

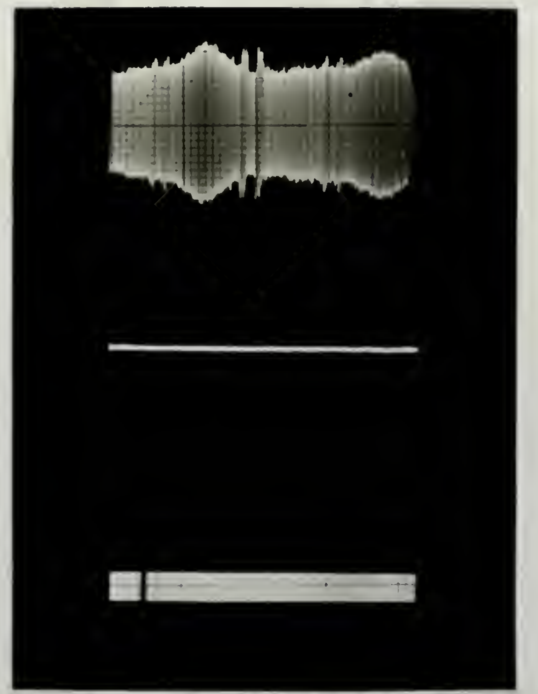


FIGURE E.5

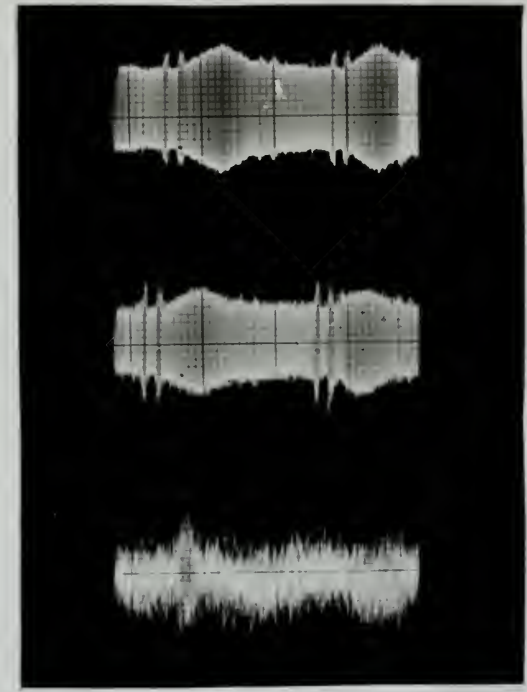


FIGURE E.6
Showing effect of air gap on signal noise,

second and the disc was being run at a speed which made this frequency look like 1950 cycles/second. The amplifier noise level and a 60 cycle/second reference are also shown. In this case the butt joint did not show up but the effect of two blisters did. These two transients were shown not to be related to the butt joint by noting in which part of the disc revolution these two transients occured and then noting when the butt joint came up to the reproduce head. Two blisters were seen to coincide with the position of these transients. The tape used for this photograph was several days old and some of the rubber cement had dried without holding the tape to the disc. In many cases no blisters were present.

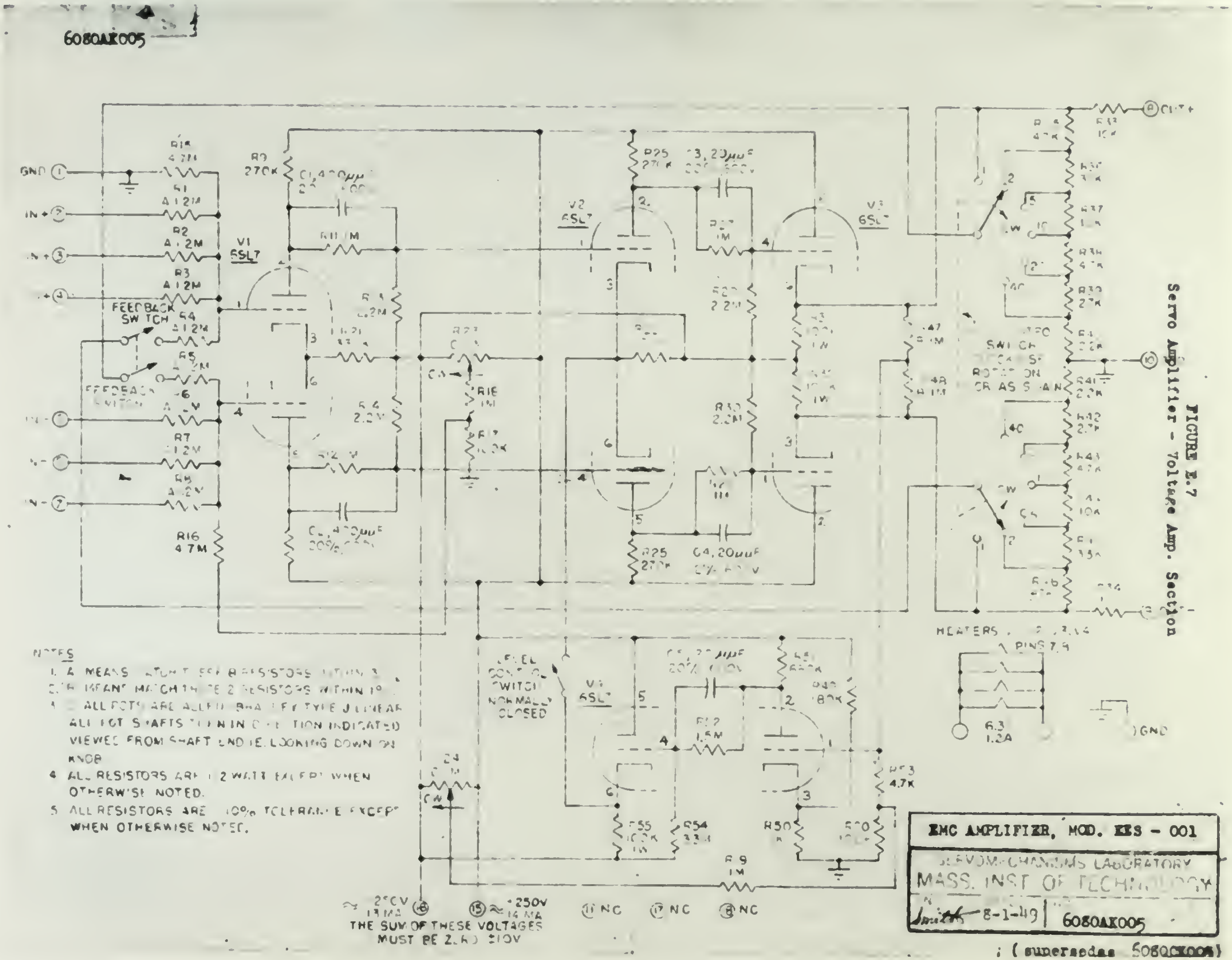
It was thought that the effect of the variation due to the eccentricity of the disc and the effect of irregularities of the tape surface might be lessened by setting the reproduce the head back from the tape. Thus the ratio of the air gap maximum to air gap minimum would be decreased. Figure E.6 shows the effect of increasing the air gap. Transients due to blisters have been accentuated rather than decreased. At a air gap of 0.017 inches the variation of the eccentricity of the disc has not been effectively smoothed. At a larger air gap setting the noise of system has overcome the signal, and it does not appear that the variation of the signal due to the eccentricity of the disc has been smoothed a great deal. In these three photographs of Figure E.6 the magnitude of the signal can not be compared since the gain of the amplifier and oscilloscope has been adjusted to give an indication which could be photographed. Actually the attenuation associated with a large air gap would require more gain and would result in a smaller signal to noise ratio.

Ward-Leonard System

The description and analysis of the Ward-Leonard system for speed

control are adequately covered in the literature.[34,35] Figure E.1 shows the arrangement by which the motor of the Ward-Leonard system drove the disc assembly. The motor used was an Oster, type E75, 115 volts, 3650 RPM, 1/20 horsepower, shunt wound. The generator of the system was the same. The amplifier used with the system is shown in schematic form in Figure E.7 and E.8.

It was originally planned to drive this system as a servomechanism with the error signal being derived by comparing the control signal to the speed of the motor as measured by a tachometer. Figure E.1 also shows a tachometer mounted with the motor. Several D.C. tachmometers were used, but all operated improperly; particularly disadvantageous was their fluctuation in terminal voltage at low speeds. It was decided to run the Ward-Leonard system open loop using the speed control characteristics of the DC motor. Because of the small variation in load troque (except at low speeds when the coulomb friction contributes to a large percentage of the load torque) this plan was thought to be feasible. The transfer function between the controlling voltage at the servo amplifier input and the speed of the disc as measured by a speed counter is given in Figure E.9. The relationship between the controlling voltage and the output of the power amplifier (the generator field current) is also shown in Figure E.9. This figure indicates that a linear relationship holds for about a speed variation of four times (from 15 RPM to 60 RPM). For a complete analyzer such a limitation would not be desirable, but for this investigation we proceeded with this limited range of variation. As it happened this limited range was not a controlling factor in the investigation. When we compared several trial runs with thesystem, it was discovered that different results were being obtained with no change in the various parameters which should control the end


6080AK005
GND ①
IN+ ②
IN+ ③
④
IN- ⑤
IN- ⑥
IN- ⑦
R15 4.7M
R1 Δ 12M
R2 Δ 12M
R3 Δ 12M
R4 Δ 12M
R5 Δ 12M
R6 Δ 12M
R7 Δ 12M
R8 Δ 12M
R16 4.7M
FEEDBACK SWITCH
FEEDBACK SWITCH
V1 6SL7
R9 270K
C1, 400μμF
R11 1M
R12 1M
R13 2.2M
R14 2.2M
V2 6SL7
R25 270K
R25 270K
C3, 20μμF
C4, 20μμF
R29 2.2M
R30 2.2M
R18 1M
V3 6SL7
R47
R48
SWITCH
CLOCKWISE ROTATION CR AS SHOWN
CW
CW
R41 22K
R42 27K
R43 47K
OUT+ ⑧
⑩
HEATERS
PINS 7,8
6.3 1.2A
GND
LEVEL CONTROL SWITCH NORMALLY CLOSED
V4 6SL7
C5, 20μμF 20%
R52 1.5M
R49 180K
R53 4.7K
R55 100K
R54 33K
R50
R51
R9 1M
R24
CW
≈ 250V 13 MA ⑯
⑮ ≈ +250V 14 MA
THE SUM OF THESE VOLTAGES MUST BE ZERO ±10V
⑪ NC
⑰ NC
⑱ NC

NOTES

1. Δ MEANS MATCH THESE 8 RESISTORS WITHIN 3%.
2. B MEANS MATCH THESE 2 RESISTORS WITHIN 1%.
3. ALL POTS ARE ALLEN BRADLEY TYPE J LINEAR ALL POT SHAFTS TURN IN DIRECTION INDICATED VIEWED FROM SHAFT END IE. LOOKING DOWN ON KNOB
4. ALL RESISTORS ARE 1/2 WATT EXCEPT WHEN OTHERWISE NOTED.
5. ALL RESISTORS ARE ±10% TOLERANCE EXCEPT WHEN OTHERWISE NOTED.

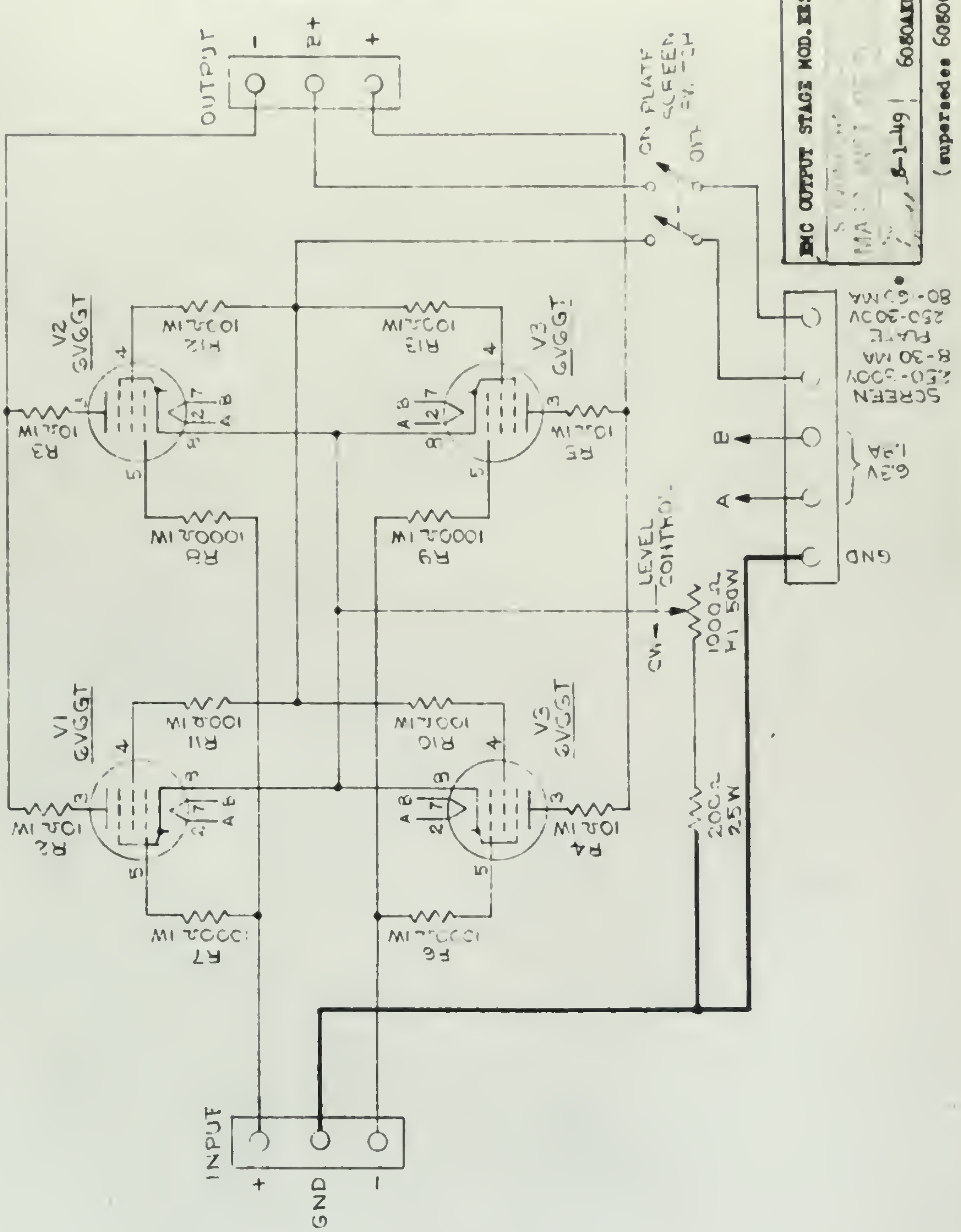
EMC AMPLIFIER, MOD. EES - 001

SERVOMECHANISMS LABORATORY
MASS. INST. OF TECHNOLOGY
Smith 8-1-49 | 6080AK005

; (supersedes 5080CK005)

FIGURE E.7
Servo Amplifier - Voltage Amp. Section

FIGURE E.8
Servo Amplifier - Power Section


EMC OUTPUT STAGE MOD.EES - 002
8-1-49
6080AK013
(supersedes 6080CK004)
OUTPUT
-
P+
+
ON PLATE
SCREEN
SW. CH
OFF
V2
6V6GT
V3
6V6GT
R12
100Ω 1W
R13
100Ω 1W
R3
10Ω 1W
R5
10Ω 1W
R8
1000Ω 1W
R9
1000Ω 1W
LEVEL
CONTROL
CW
1000 Ω
H1 50W
V1
6V6GT
V3
6V6GT
R11
100Ω 1W
R10
100Ω 1W
R2
10Ω 1W
R4
10Ω 1W
R7
1000Ω 1W
R6
1000Ω 1W
200Ω
25W
SCREEN
250-300V
8-30 MA
PLATE
250-300V
80-150 MA
6.3V
1.4A
A
B
GND
INPUT
+
GND
-

6080AK013

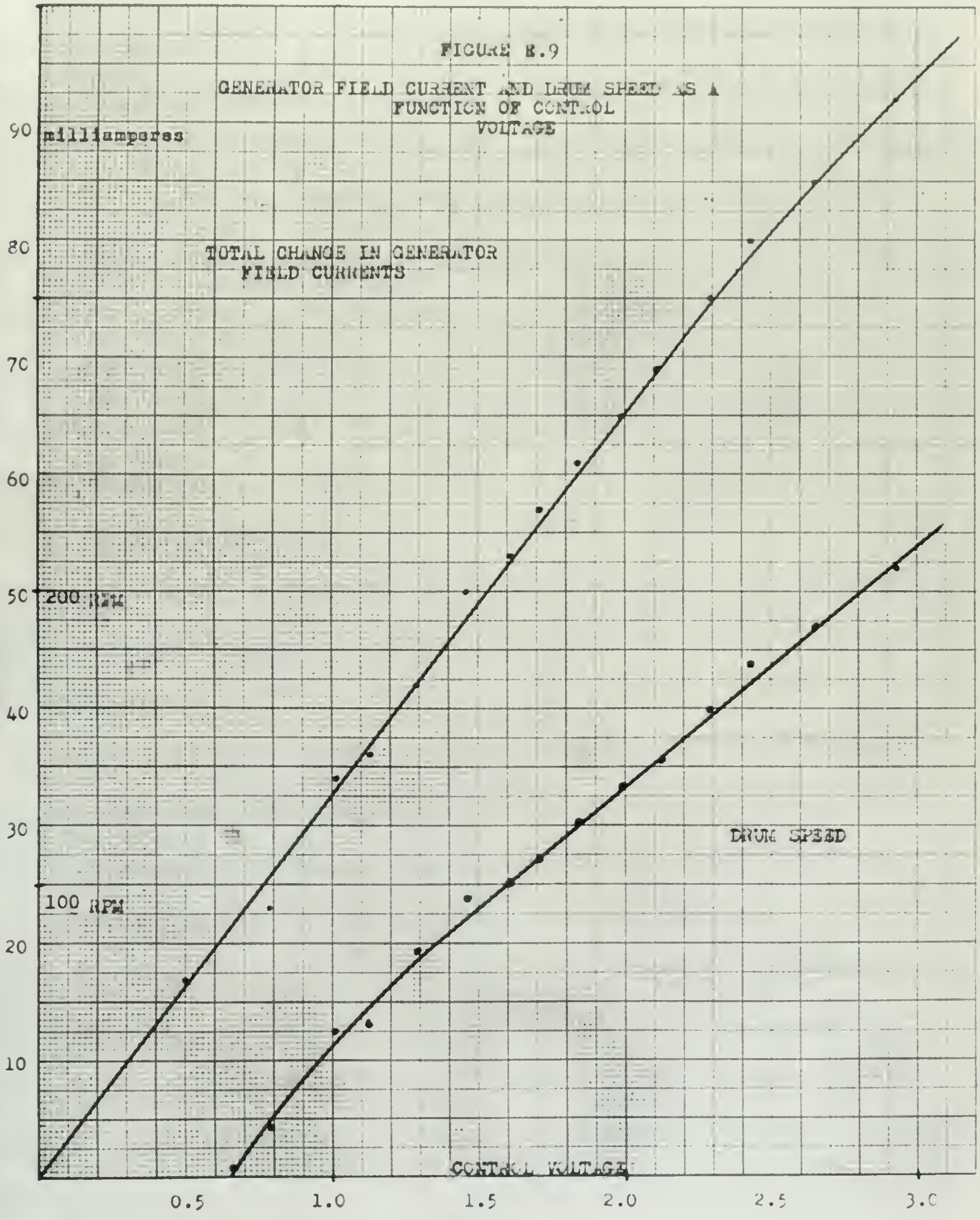


FIGURE B.9

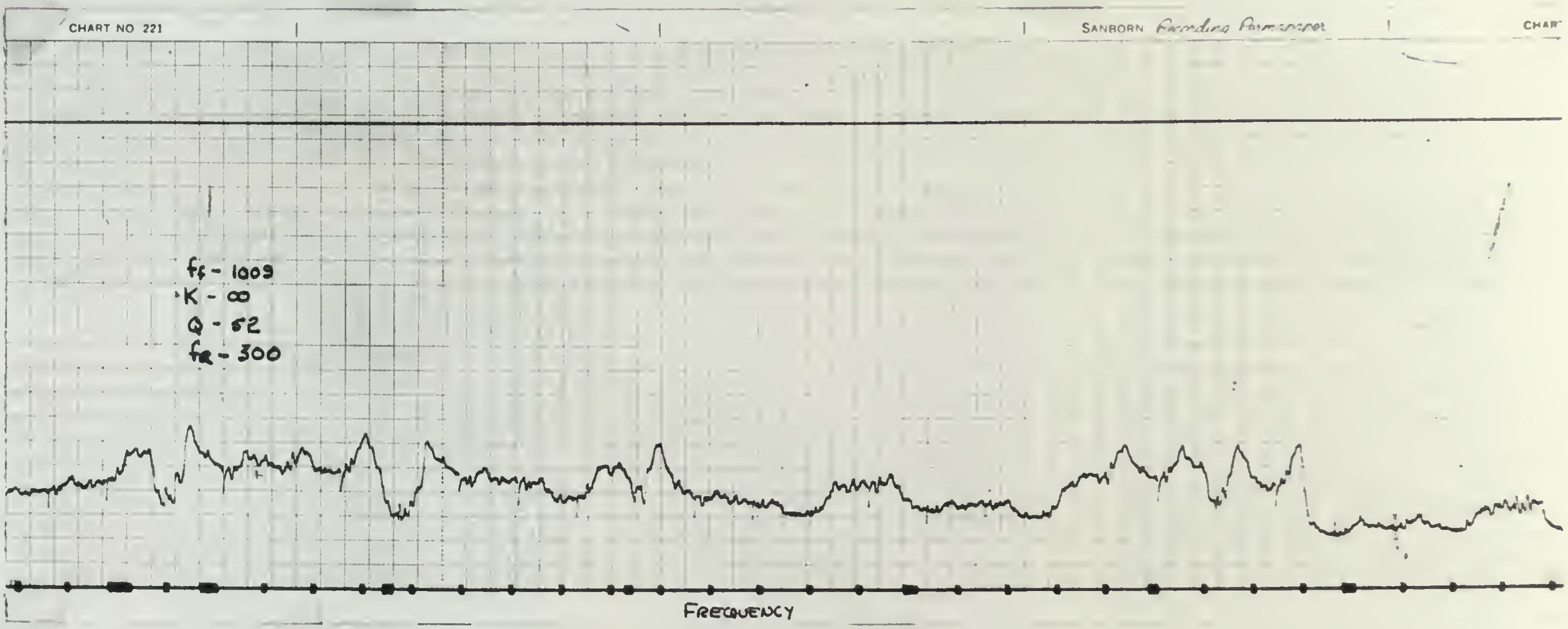
GENERATOR FIELD CURRENT AND DRUM SPEED AS A FUNCTION OF CONTROL VOLTAGE

results. Figure E.11 shows the results of three different runs for a recorded frequency of 300 cycles per second, Q of 22. It was thought that the differences between the three indications were due to fluctuations in the speed of the disc as it was slowed down by the controlling voltage. Therefore a test was made with a constant controlling voltage. Figure E.10 shows the results of this test. Ideally a constant controlling voltage would result in a constant disc speed and a consequent steady output voltage. With the variation of the signal due to the eccentricity of the disc the output indication would necessarily be periodic. The results of this variation can easily be seen in Figure E.10. However, in addition to this periodic variation there is an erratic variation of the output which can only be explained as the erratic variation in input frequency to the tuned circuit. That this variation was due to the signal coming from the magnetic tape was shown by obtaining a steady response from the selective system and detector using a steady signal from an audio oscillator.

In order to obtain a smoother variation of the disc speed it was decided to disconnect the disc from the driving motor and having sped the disc up to speed, (using a cord) to let the disc slow down freely. Due to the large moment of inertia of the disc, the disc should act as a filter for small variations in torque such as that from the bearings on the disc shaft. A smoother variation was obtained as is indicated by the pulses of Figure 3.9. We no longer had control of the speed of the disc (a variable damping arrangement could control the free running speed somewhat), but control of the parameter K was possible by changing the Q of the tuned circuit. The speed-time relationship for the free

FIGURE 2.10

Variation of filter output with constant controlling voltage



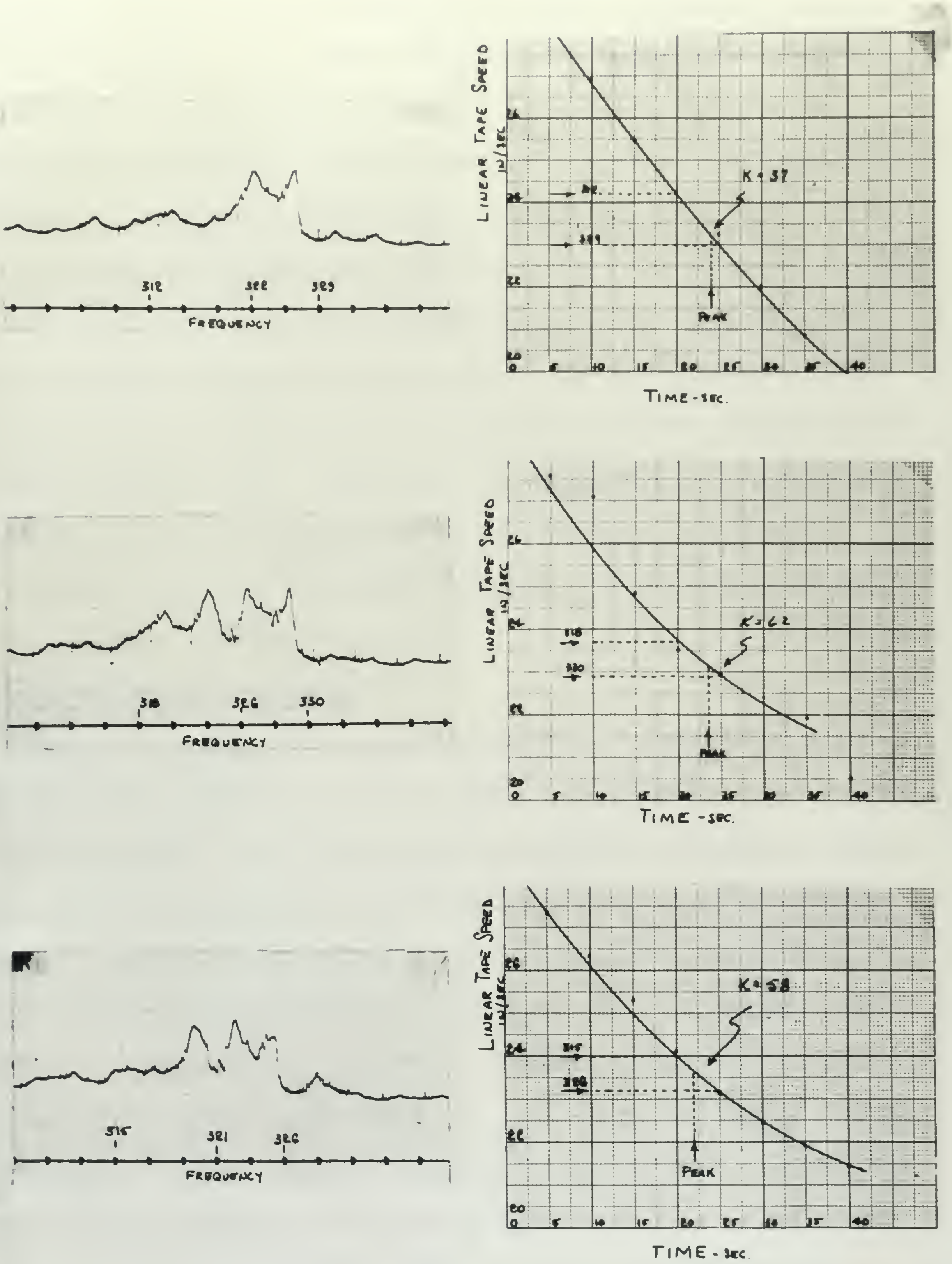


FIGURE 5.11

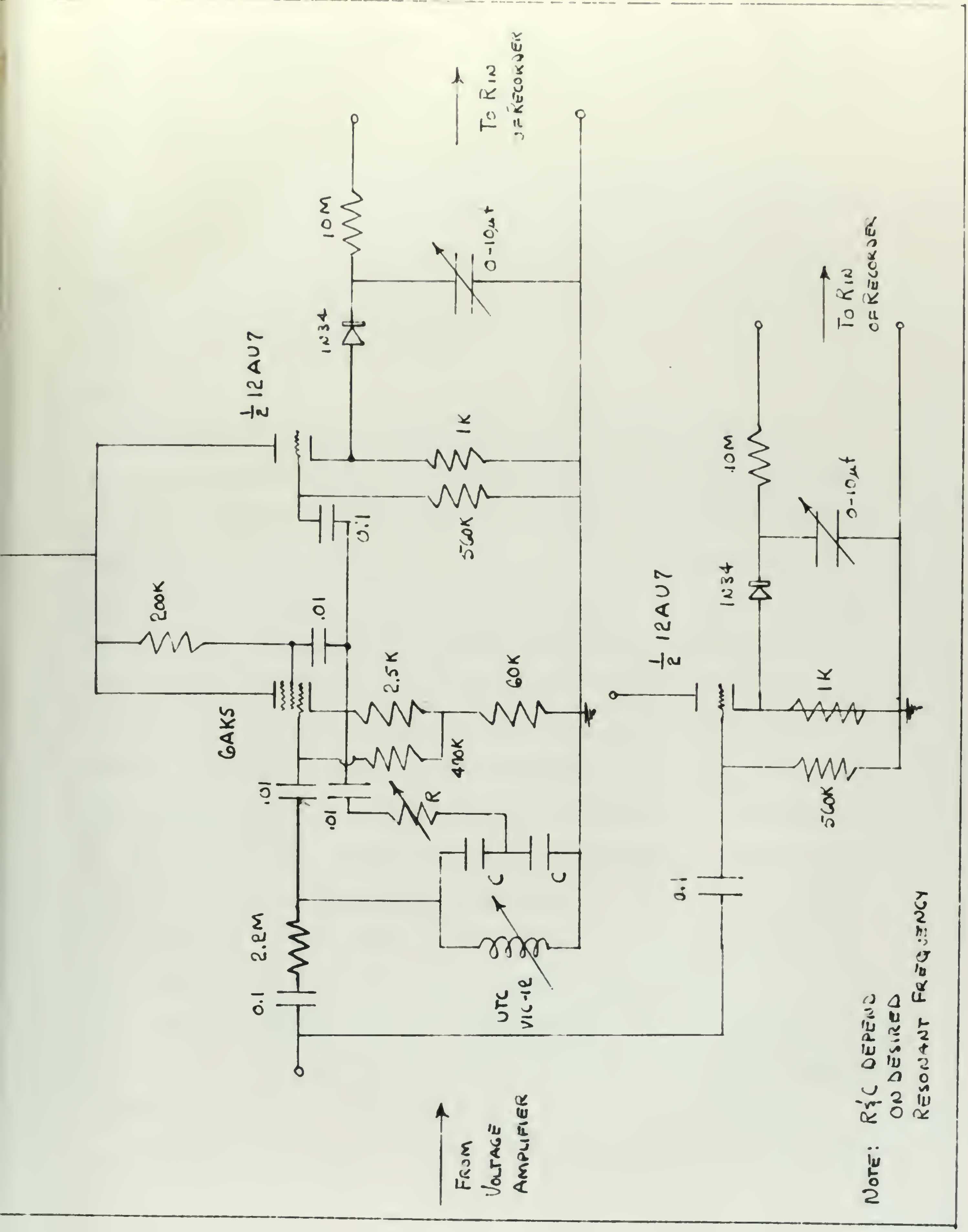
Variation of detected filter output for three different runs. All parameters remaining constant. Exponential frequency sweep.

running disc was determined by using the butt joint transient as a marker. Figure 3.9 also shows, in the middle recording, the marker recording at the top. The bottom trace shows one second marker pulses which were generated by the recorder (A Sanborn Cardette) These one second intervals were checked by counting the number over a measured period of time. The actual speed-time relationship was found by measuring the intervals between butt joint transients using a pair of dividers, and then calculating the speed of the disc knowing the circumference and the calibrated time scale of the recording. This was done for a number of points and a smooth curve was drawn through the points. The speed time relationships as determined in this manner are shown on Figures E.11, 3.8, and 3.9. The curve of figure E.11 were due to an exponential control voltage but were obtained using the procedure described above.

Selective System and Detector

A schematic of the selective System and detector is shown in Figure E.12. A parallel resonant LRC circuit with Q multiplication was used as the selective system. The Q of the resonant circuit is effectively multiplied by positive feed back from the vacuum tube which in effect cancels part of the resistance associated with the tank circuit. This circuit has good stability desired for a measurement circuit. The output of the Q multiplier is taken across the cathode resistor. A cathode follower isolation stage followed the Q multiplier. This stage was used to isolate the Q multiplier from the detector since any variations across the cathode resistor of the Q multiplier are reflected back to the tank circuit. The detector was a linear detector the design of which is covered in most elementary electronics textbooks. The decay time constant of this detector was determined by trial and error since the shape of the modulated signal

FIGURE E.12
Q-Multiplier and Detectors


From Voltage Amplifier
0.1
2.2M
UTC VIC-12
C
C
R
.01
.01
470K
6AK5
2.5K
60K
.01
200K
0.1
560K
1K
½ 12AU7
1N34
0-10μf
10M
To R IN of Recorder
½ 12AU7
0.1
560K
1K
1N34
0-10μf
10M
To R IN of Recorder
Note: R & C depend on desired resonant frequency

was not known with any exactitude. This time constant was set to show any fluctuations in the envelope of the curve due to the transient response of the filter.

The detector for the butt joint markers was of similar design. The input of thid detector was taken off the amplified reproduced signal since it was desired to mark the recording for the entire sample length and not just for those time intervals when a reproduced frequency was equal to the frequency of the tuned circuit.

Amplifier

The amplifier used was a General Radio type 714A voltage amplifier. The gain is adjustable between 20 to 80 decibels. The noise level of the amplifier in comparison to the amplified reproduced signal can be seen in Figure E.5. (It must be remembered that this signal is attenuated due to the air gap between the head and the tape in the order of 33 decibels from the 0.004 volts maximum possible. This amplifier has a flat frequency response between 20-18,000 cycles. The amplifier was very susceptible to vibrations when operated near its maximum voltage gain (A horn honking outside would produce a large output) The amplifier was removed from the table holding the drum assembly and the top was removed from the amplifier. The top was acting like a baffle and was being capacitively coupled to the grids of the tubes which were only a half inch or so below the top. (No more trouble was had with horns) The vibrations of the table carring the Ward-Leonard generator and associated motor was excessive even in that they caused the reproduce head to vibrate noticably. This assembly was moved to another table and both assemblies were put on rubber sheets which reduced the vibrations markedly.

Frequency Measurements

A General Radio frequency meter, type 1141-A, was used as a master frequency standard. This meter is of the null detector type and requires use of head phones to determine the null. The manufacturer claims that the meter is accurate within 1%. No check was made of this frequency meter. Within the range of interest, 200 cycles to 5000 cycles, the null point could be detected to the accuracy of an imperceptible movement of the dial. However, the accuracy of the measurement was limited by the accuracy with which the dial could be read. Up to 500 cycles this accuracy was estimated to be about 1 cycle.

A General Radio direct reading frequency meter, type 834-B, was used for rapid measurements in order to center in on the measurement by the null type meter.

The Q of the tuned circuit was measured by using an audio oscillator, an oscilloscope, and the frequency meter. The bandwidth of the system was taken simply as the frequency difference between the half-power points. No response curve was taken for the tuned circuit but it was noted that the curve was unsymmetrical about the maximum point.

Possible Equipment

(a) Disc and reproduce head. This assembly is plagued by the trouble experienced by most workers with drums of this sort - the error introduced by the necessity of having a varying air gap due to the eccentricity of the drum. At the same time the irregularities of the surface would cause trouble even if there were no eccentricity. There is the possibility of using a loop of tape which would be carried across the reproduce head similar to a conventional tape recorder. The fast rewind speeds of tape recorders show that tapes can be transported across the head at speeds several times greater

than normal 7.5 inch per second speed. However, there is probably some limit to the range of speeds available from this method due to breaking of the tape. Other possibilities exist such as some mechanism which would keep the head in intimate contact with the tape on the disc or drum by means of spring loading. With the widespread use of magnetic drums and the fact that this problem is common to almost all users of such drums, we hesitate to make any comments other than that this field should be investigated for some simple, practical and workable solution which has already proved itself.

(b) Driving assembly. This investigation showed that there is a real need for an accurate speed controlling mechanism. At first one would immediately think of a feedback system; however the application in this case is one which does not require the advantage of the feedback servomechanism in maintaining speed control with widely varying torques . In this application it is important that the instantaneous speed of the disc not vary within limits, and these limits are set by the resolution of the system. Any feedback system would require that the error signal correct the speed before the recorded frequency being analyzed at that instant pass out of the pass band of the filter. Further investigation would be necessary to determine whether this feedback system would prove satisfactory. The possibility of the freely running drum as was used in this investigation shows promise in that the speed of the drum varies smoothly and that additional damping could be introduced to control the rate of slowing down. However, such a proposal would require a marker system coupled with some additional means of providing a direct reading indication of the analyzed frequency. The method as used here to determine the frequency of the sample is too laborious for a practical system.

(c) Function generator. A function generator which works off a function masked on the face of an oscilloscope was obtained but was not used. This function generator is in wide use today and has proved itself. For this investigation which was limited to exponential and free running variation of speed, the generator was not needed. For exponential or linearly varying voltages a simple RC decay or sweep circuit generator are recommended. The use of cams is possible, and these cams were investigated. However, precision cams followed by a translatory potentiometer would cost as much as a functionally wound potentiometer. For the function necessary for the equal sample analysis a function wound potentiometer and constant speed drive is recommended.

APPENDIX F

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